Optimal Controlled Phasegates for Trapped Neutral Atoms at the Quantum Speed Limit

Michael Goerz*
Tommaso Calarco†
Christiane P. Koch*

*Universität Kassel
†Universität Ulm

DPG Spring Meeting
Dresden
March 16, 2011

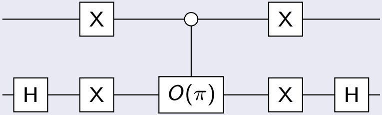
Universal Quantum Computing

Controlled Phasegate

$$\hat{\mathbf{O}}(\chi) = \text{CPHASE}(\chi) = \begin{pmatrix} e^{i\chi} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

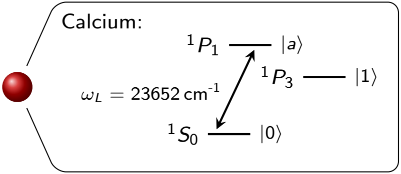
Controlled-Not

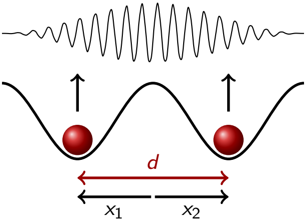
CNOT =



- CPHASE(π) equivalent to CNOT $\Rightarrow$ Universal Quantum Computing
- CPHASE is used in Quantum Fourier Transform

Two-Qubit Gates on Trapped Neutral Atoms





- Low-Lying states in Alkaline-Earth atoms or Rydberg states
- Atoms in optical lattice or optical tweezers

The Objective

Problem

- QC with atomic collisions: adiabaticity $\Rightarrow$ slow.
- Strong interaction $\Rightarrow$ fast gates?
  – only if ignoring motion.

Quantum Speed limit

- QSL: What is the maximum speed at which a quantum system can evolve?
- What limits on the gate duration can we find through optimization?
- How do gate durations depend on the interaction strength?

The Objective

Problem

- QC with atomic collisions: adiabaticity $\Rightarrow$ slow.
- Strong interaction $\Rightarrow$ fast gates?
  – only if ignoring motion.

Quantum Speed limit

- QSL: What is the maximum speed at which a quantum system can evolve?
- What limits on the gate duration can we find through optimization?
- How do gate durations depend on the interaction strength?

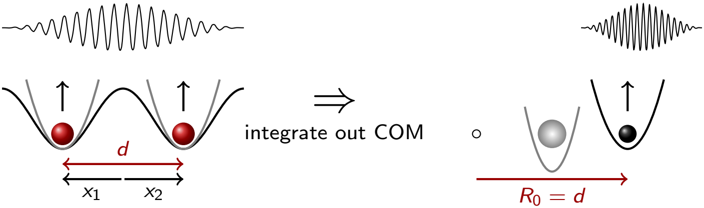
Approach

- Describe the system including the motional degree of freedom.
- Optimize for varying times / interaction strengths:
  - I Two Calcium atoms at fixed distance (fixed interaction): vary T
  - II For fixed T, two atoms with "artificial" dipole-dipole interaction $V(R) = -C_3/R^3$: vary C_3

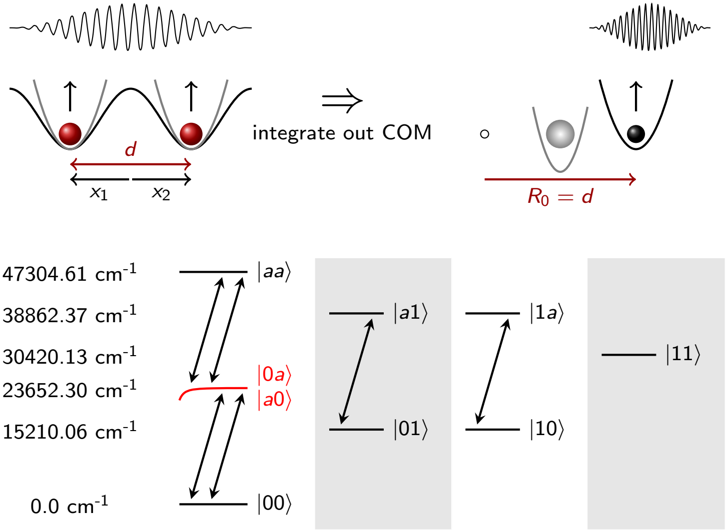
Theoretical Model and Optimization Method

Two-Qubit-Hamiltonian, Optimization with Krotov

System Hamiltonian



System Hamiltonian



Optimizing the Laser Pulse

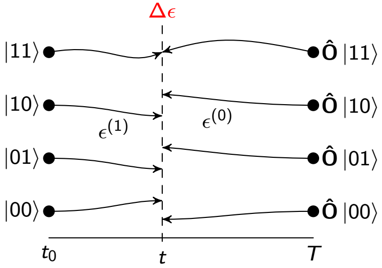
Target Functional

$$J = \underbrace{-\frac{1}{N}\mathfrak{Re}\left[\mathrm{tr}\left(\hat{\mathbf{O}}^\dagger \hat{\mathbf{U}}\right)\right]}_{F} + \int_0^T \frac{\alpha}{S(t)} \Delta\epsilon^2(t)\,\mathrm{d}t; \qquad \begin{aligned} \hat{\mathbf{O}} &= \text{CPHASE} \\ \hat{\mathbf{U}} &= e^{-i\hat{\mathbf{H}}(\epsilon(t))t} \end{aligned}$$

Krotov: pulse update $\Delta\epsilon$ minimizing J

$\Delta\epsilon \sim \mathfrak{Im}\left\langle \Psi_{bw} \left| \hat{\boldsymbol{\mu}} \right| \Psi_{fw} \right\rangle$

Palao, Kosloff,
PRA 68, 062308 (2003)



Measures of Merit

Fidelity F and cost functional J are not very informative.

Control over the Motional Degree of Freedom

$$F_{00} = \left| \left\langle 00(R) \middle| \hat{\mathbf{U}}(T, 0; \epsilon^{opt}) \middle| 00(R) \right\rangle \right|^2$$

Does $|00\rangle$ return to it's initial **vibrational eigenstate**?

Gate Phases

$$\phi_{00} = \arg\left(\left\langle 00(R) \middle| \hat{\mathbf{U}}(T, 0; \epsilon^{opt}) \middle| 00(R) \right\rangle \right)$$

What is the **phase change** relative to the initial state?

True Two-Qubit Phase

Cartan Decomposition leads to $\chi = \phi_{00} - \phi_{01} - \phi_{10} + \phi_{11}$

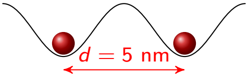
Concurrence (Entanglement) $C = \left| \sin \frac{\chi}{2} \right|$

Two Calcium Atoms at Short Internuclear Distance

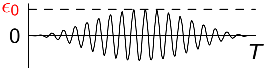
For which gate durations can we reach a high-fidelity CPHASE?

Parameters of the Optimization

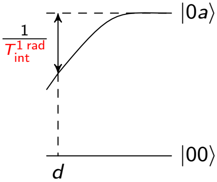
- Short internuclear distance
  $\Rightarrow$ sufficient interaction

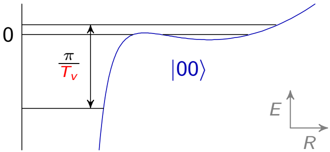


- Peak intensity ϵ_0
  to induce 1 Rabi cycle

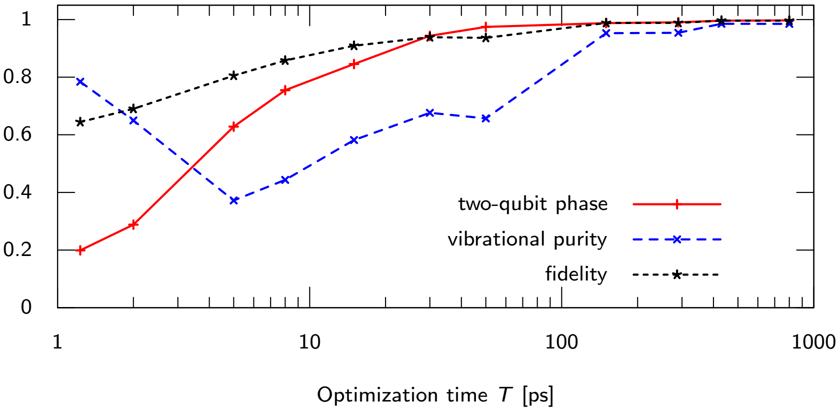


- Pulse duration between $T_{\text{int}}^{1\,\text{rad}} = 1.23$ ps and $T_v = 800$ ps





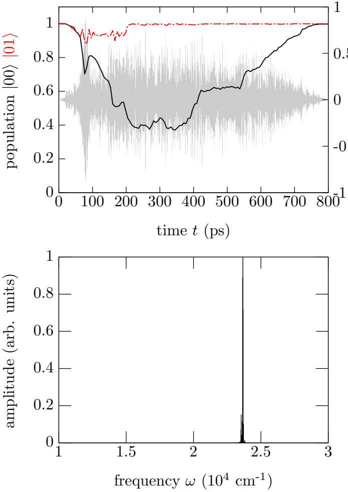
Optimization Success over Pulse Duration



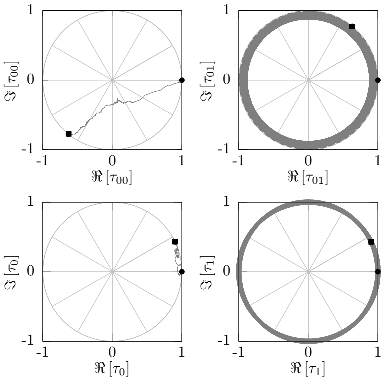
$\Rightarrow$ For small T, vibrational purity is lost with increasing two-qubit phase

$\Rightarrow$ High two-qubit phase *and* high vibrational only for long pulse durations

System Dynamics for 800 ps Pulse



$F = 0.997$



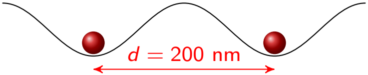
$$\tau_{00} = \left\langle 00(R) \middle| \hat{\mathbf{U}}(T, 0; \epsilon^{opt}) \middle| 00(R) \right\rangle$$

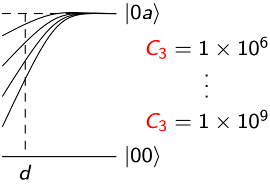
Two Atoms at Long Distance under Strong Dipole-Dipole Interaction

Can we avoid vibration with very short pulses, but very strong interaction?

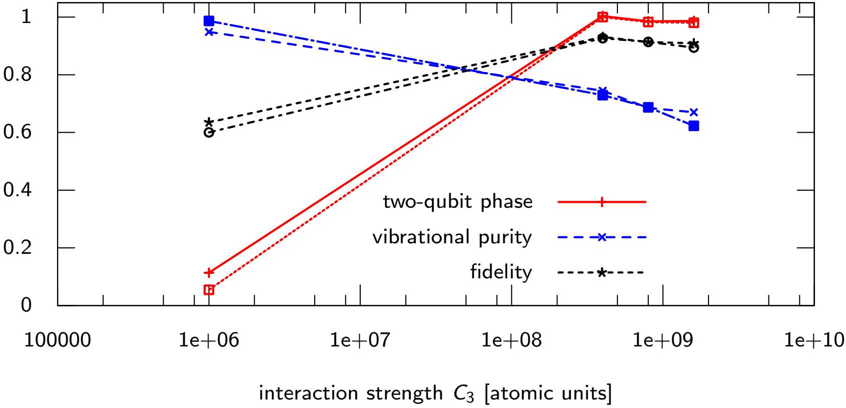
Parameters of the Optimization

- Fixed short pulse duration
  $T = 1$ ps, $T = 0.5$ ps
- Realistic lattice spacing
  with strong interaction $\sim -\frac{C_3}{R^3}$
- Vary C_3:
  - $C_3 = 1 \times 10^6$
    Action over 1 ps for Calcium at
    $d = 5$ nm, scaled to $d = 200$ nm
  - Increase by three orders of magnitude
    Action over 800 ps for Calcium at
    $d = 5$ nm, scaled to $d = 200$ nm





Optimization Success over Dipole Interaction Strength

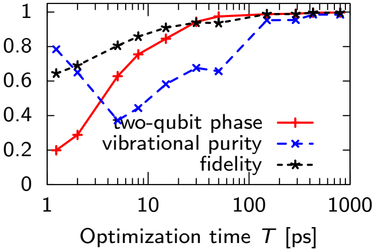


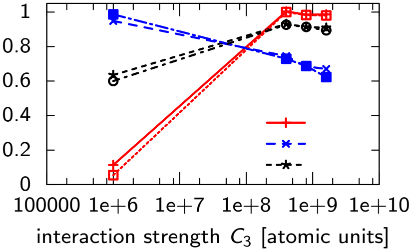
⇒ Increasing two-qubit-phase with increasing interaction strength

⇒ For small T, vibrational purity is lost with increasing two-qubit phase

Conclusions

Conclusions





- Long gate duration can reach arbitrarily high fidelities.
- For short gate durations, the two-qubit phase is at the expense of the vibrational purity.
- If $T <$ QSL, not all measures of merit can be fulfilled.
- Time scale for a successful gate is determined by $\max(T_{int}, T_{vib})$.

Acknowledgements

AG Koch

- Christiane Koch
- Daniel Reich
- Mamadou Ndong
- Ruzin Ağanoğlu
- Giulia Gualdi
- Anton Haase
- Martin Berglund

Funding

Financial support from the Deutsche Forschungsgemeinschaft is gratefully acknowledged (Grant No. KO2302)