

# Experiment

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Date: 7.3.05

Tutor: Lisong Yang

Begin: 10<sup>20</sup>

End: 13<sup>10</sup>

## Assignment B1:

Devices:

- VOLTCRAFT VC220 (multimeter):  $U: 0,6\% + 5d$ ;  $R: 1\% + 3d$   
 $I: 1\% + 2d$
- resistors:

label	measured value
3,3 $\Omega$	3,5 $\Omega \pm 0,4 \Omega$
6,8 $\Omega$	5,9 $\Omega \pm 0,4 \Omega$
12,0 $\Omega$	12,3 $\Omega \pm 0,5 \Omega$
27,0 $\Omega$	27,5 $\Omega \pm 0,6 \Omega$
56,0 $\Omega$	62,8 $\Omega \pm 0,9 \Omega$

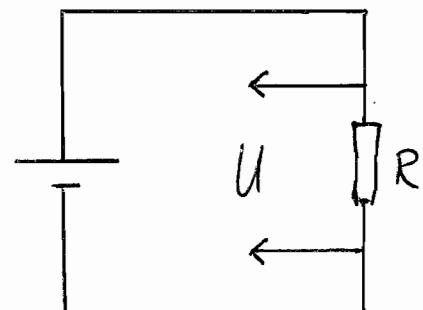
- battery: voltage  $U_0 = (1,523 \pm 0,013) V$

First measurement: (battery)

resistor ( $\Omega$ )	voltage (V)
3,5 $\pm 0,4$	1,368 $\pm 0,014$
3,5 $\pm 0,4$	1,358 $\pm 0,014$
5,9 $\pm 0,4$	1,428 $\pm 0,014$
5,8 $\pm 0,4$	1,430 $\pm 0,014$
12,3 $\pm 0,5$	/
12,3 $\pm 0,5$	
27,5 $\pm 0,6$	/
27,5 $\pm 0,6$	
62,8 $\pm 0,9$	/
62,8 $\pm 0,9$	

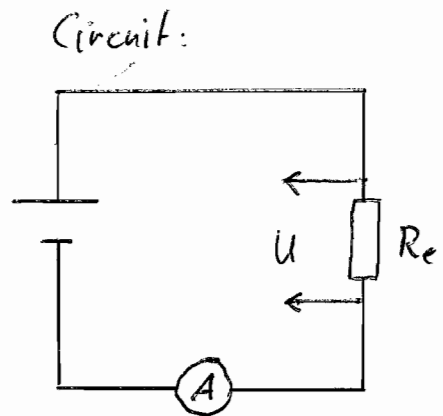
discussion  
later on

Circuit:



## Second measurement: (battery)

resistor ( $R_e$ ) $\Omega$	voltage V	current A
$3,5 \pm 0,4$	$1,341 \pm 0,018$	$0,189 \pm 0,003$
$5,9 \pm 0,4$	$1,418 \pm 0,014$	$0,23 \pm 0,005$
$12,3 \pm 0,5$	$1,463 \pm 0,014$	$0,12 \pm 0,005$
$27,5 \pm 0,6$	$1,363 \pm 0,014$	$0,0498 \pm 0,0010$
$62,8 \pm 0,9$	$1,447 \pm 0,014$	$0,0219 \pm 0,0005$



Measurement for power supply (the circuit is equal to the circuit of the second measurement of the battery):

$$U_0 = (2,00 \pm 0,07) \text{ V}$$

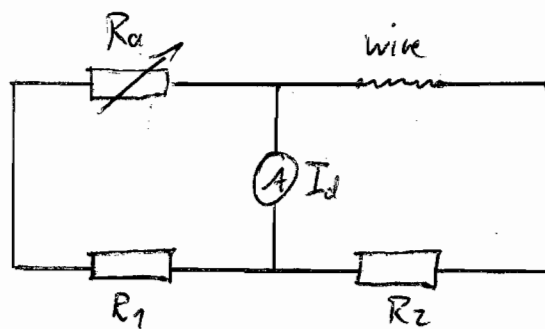
resistor ( $R_e$ ) $\Omega$	voltage V	current A
$3,5 \pm 0,4$	$1,947 \pm 0,017$	$0,57 \pm 0,005$
$5,9 \pm 0,4$	$1,969 \pm 0,017$	$0,32 \pm 0,003$
$12,3 \pm 0,5$	$1,983 \pm 0,017$	$0,1353 \pm 0,0016$
$27,5 \pm 0,6$	$1,823 \pm 0,016$	$0,0668 \pm 0,0009$
$62,8 \pm 0,9$	$1,920 \pm 0,017$	$0,0296 \pm 0,0005$

## Assignment C1:

### Devices:

- VOLTcraft VC 220:  $R: 1\% + 3d$ ;  $I: 1\% + 2d$
- resistors:
  - $R_1 = (100,8 \pm 1,3) \Omega$  label:  $(100,5 \pm 0,1) \Omega$
  - $R_2 = (0,132 \pm 0,001) \Omega$  (label only)
- copper wire, length:  $(1,000 \pm 0,005) \text{ m}$ ,  $\phi = 0,2 \text{ mm} \pm 5\%$
- adjustable resistor, range:  $0 - 1000 \Omega$  (called  $R_a$ )

### Circuit:



### Measurement:

$R_a (\Omega)$	$I_d (A)$
$443 \pm 8$	$0,00 \pm 0,02$
$441 \pm 8$	$0,00 \pm 0,02$

# Assignment F1:

Devices:

- R-C-L multimeter: capacity:  $0,7\% + 3d$
- VOLTcraft VC220: R:  $1\% + 3d$
- OSCILLOSCOPE HM 103
- frequency generator VOLTcraft F202
- capacitors:  $C_1 = (0,0997 \pm 0,0010) \mu F$   
 $C_2 = (1,007 \pm 0,010) \mu F$
- resistors:  $R_1 = (18,00 \pm 0,5) k\Omega$   
 ~~$R_2 = (804 \pm \dots)$~~  unused!

Measurement:

capacitor  $C_1$  resistor  $R_1$

I)	voltage (V)	time (ms)	II)	voltage (V)	time (ms)
	$5,6 \pm 0,4$	0		$6,0 \pm 0,4$	0
	4,8	0,5		4,8	0,5
	3,4	1,0		3,2	1,0
	2,8	1,5		2,0	1,5
	2,0	2,0		1,6	2,0
	1,6	2,5		1,2	2,5
	1,2	3,0		1,0	3,0
	0,8	3,5		0,8	3,5
	0,6	4,0		0,6	4,0
	0,4	4,5		0,4	4,5

capacitor  $C_2$  resistor  $R_1$ :

time (ms)	voltage (V)	
0	$2,0 \pm 0,2$	all values + 2V !
1	1,7	
2	1,5	
3	1,3	
4	1,1	
5	0,9	
6	0,7	
7	0,5	
8	0,3	
9	0,2	

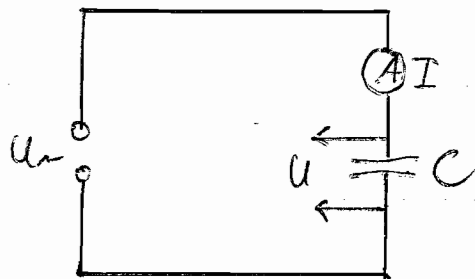
The measurement with  $R_2$  results into a very slow discharge of the ~~off~~ capacitor.

### Assignment G1:

Devices:

- VOLTcraft VC220  $\mathbb{R}$   $U: 0,6\% + 5d$  ;  $I: 1\% + 2d$
- capacitor:  $C = (1,007 \pm 0,010) \mu F$
- frequency generator

Circuit:



Measurement:

frequency (Hz)	voltage (V)	current (mA)
20 ± 1	3,38 ± 0,07	0,0375 ± 0,0006
50 ± 1	3,29 ± 0,07	0,0871 ± 0,0010
100 ± 1	3,19 ± 0,07	0,1602 ± 0,0026
200 ± 1	3,22 ± 0,07	0,800 ± 0,010
500 ± 1	2,93 ± 0,07	1,948 ± 0,030
1000 ± 1	2,47 ± 0,07	12,15 ± 0,20
1500 ± 1	2,55 ± 0,07	21,1 ± 0,5
2000 ± 1	2,28 ± 0,07	26,7 ± 0,5
3013 ± 1	1,79 ± 0,06	35,3 ± 0,6
4008 ± 1	1,42 ± 0,06	41,9 ± 0,7
5010 ± 1	1,14 ± 0,06	45,6 ± 0,7
6002 ± 1	0,91 ± 0,06	47,8 ± 0,7
10072 ± 1	0,31 ± 0,06	48,9 ± 0,2

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## Analysis

On Assignment B1:

Our first measurement was based on the following equations:

$$R_i = \frac{U_0 \cdot R_e}{U} - R_e$$

which is a transformation of the formula presented in the introduction. The problem of this measurement is the accuracy of the external resistor. The error of 12% for example causes a high inaccuracy of the tiny internal resistance (first calculations during the measurement resulted into a relative error of more than 100%). Therefore we aborted this experiment.

Another way of calculation could be found using Ohm's law. The total voltage could only be reached considering both resistances as follows:

$$\begin{aligned} U_0 &= (R_i + R_e) \cdot I \\ \Rightarrow U_0 &= R_i \cdot I + R_e \cdot I \\ \Rightarrow U_0 &= R_i \cdot I + U \\ \Rightarrow R_i &= \frac{U_0 - U}{I} \end{aligned}$$

This equation is independent from  $R_e$  (according to the measurement) and bypasses the high error.

In this case the error is sufficiently low, such that the values can be calculated in the following table.

First measurement: battery  $\Rightarrow U_0 = (1,523 \pm 0,013) \text{ V}$

measured value for $R_e$	calculated value of $R_i$ ( $\Omega$ )
$(3,5 \pm 0,4) \Omega$	$0,47 \pm 0,18$
$(5,9 \pm 0,4) \Omega$	$0,46 \pm 0,12$
$(12,3 \pm 0,5) \Omega$	$0,50 \pm 0,14$
$(27,5 \pm 0,6) \Omega$	$3,21 \pm 1,06$
$(62,8 \pm 0,9) \Omega$	$3,47 \pm 0,56$

As we expected in the introduction, the internal resistance depends on the load of the battery. The "jump" between  $R_e = (12,3 \pm 0,5) \Omega$  and  $R_e = (27,5 \pm 0,6) \Omega$  is hard to explain and more external resistors would be necessary to get sufficiently measurements for an accurate proposition.

The following table was used for calculation:

U_0	Error	U_R	Error	I_R	Error	R_i
1.523	0.013	1.341	0.014	0.39	0.03	$0.47 \pm 0.18$
1.523	0.013	1.418	0.014	0.23	0.03	$0.46 \pm 0.12$
1.523	0.013	1.463	0.014	0.12	0.03	$0.50 \pm 0.14$
1.523	0.013	1.363	0.014	0.0498	0.0010	$3.21 \pm 1.06$
1.523	0.013	1.447	0.014	0.0219	0.0005	$3.47 \pm 0.56$

Second measurement: (virtual) internal resistance of a power supply

The circuit used was the same as above. Again voltage and current have been measured. In this case, we got a dependency of the internal resistance on the external load of the power supply, as well. The voltage  $U_0$  has been dedicated as follows:

$$U_0 = (2,00 \pm 0,07) V$$

From this follows a nearly similar calculation table:



U <sub>0</sub>	Error	U <sub>R</sub>	Error	I <sub>R</sub>	Error	R <sub>i</sub>
2.00	0.07	1.947	0.017	0.57	0.03	0.09 ± 0.02
2.00	0.07	1.969	0.017	0.32	0.03	0.10 ± 0.02
2.00	0.07	1.983	0.017	0.1359	0.0016	0.13 ± 0.01
2.00	0.07	1.823	0.016	0.0668	0.0009	2.65 ± 1.30
2.00	0.07	1.920	0.017	0.0296	0.0005	2.70 ± 0.70

Therefore the table of the relationship between  $R_e$  and  $R_i$  is given as:

measured value of $R_e$ ( $\Omega$ )	calculated value of $R_i$
3,5 ± 0,4	0,09 ± 0,02
5,9 ± 0,4	0,10 ± 0,02
12,3 ± 0,5	0,13 ± 0,04
27,5 ± 0,6	2,65 ± 1,30
67,8 ± 0,5	2,70 ± 0,70

The most interesting part of this table, is that we have a "jump" between  $R_e = (12,3 \pm 0,5) \Omega$  and  $R_e = (27,5 \pm 0,6) \Omega$ , as well. Therefore this might be either a systematic error during the measurement or a characteristic behavior of the internal resistance of a power supply and a battery. But as already mentioned, the insufficient quantity of measured values does not allow a more accurate proposition.

On Assignment C1:

The intention of this experiment was to measure the resistance (which was expected to be very small) of a copper wire. The technique used was a Wheatstone bridge as described in the introduction.

By regulating the adjustable resistance we got three parameters, which made the unknown resistance of the copper wire calculable with high precision:

$$R_w = \frac{R_a \cdot R_2}{R_1}$$

From this follows:

first:  $R_w = (0,58 \pm 0,02) \Omega$

second:  $R_w = (0,58 \pm 0,02) \Omega$

result:  $R_w = (0,58 \pm 0,02) \Omega$

The specific resistance can be calculated as follows:

$$r_c = \frac{R_w \cdot A}{l}$$

The length and area are given as:  $l = (1,000 \pm 0,005) \text{ m}$

$$A = (0,031 \pm 0,002) \text{ mm}^2$$

$$= (0,031 \pm 0,002) \cdot 10^{-6} \text{ m}^2$$

Now the specific resistance can be calculated, with the following result:

$$r_c = (1,80 \pm 0,20) \cdot 10^{-8} \Omega \text{ m}$$

This value is equal to the literature value of  $r_c = 1,7 \cdot 10^{-8} \Omega \text{ m}$ , which is another confirmation for the accuracy of the measurement.

On Assignment F1:

During the measurement with the oscilloscope, an exponential decrease was displayed. This is in correspondence to ~~the~~ what we expected. Therefore the following plots have a half-logarithmic scale to determine the exponent of the exponential function. The two measurements with the capacitor  $C_1$  are plotted on the following two pages. All further calculations on this page correlate on these plots, labeled "Measurement I" and "Measurement II".

On Measurement I:

The slope of the fit-plot can be calculated as follows:

$$m_I = - \frac{\ln(5/0,4)}{(5,1 - 0,3)}$$

Considering the error-fit we get:  $m_I = -(526 \pm 30) \frac{V}{s}$

On Measurement II:

In this case we get:

$$m_{II} = -(523 \pm 95) \frac{V}{s}$$

The average value of measurement I and II is:

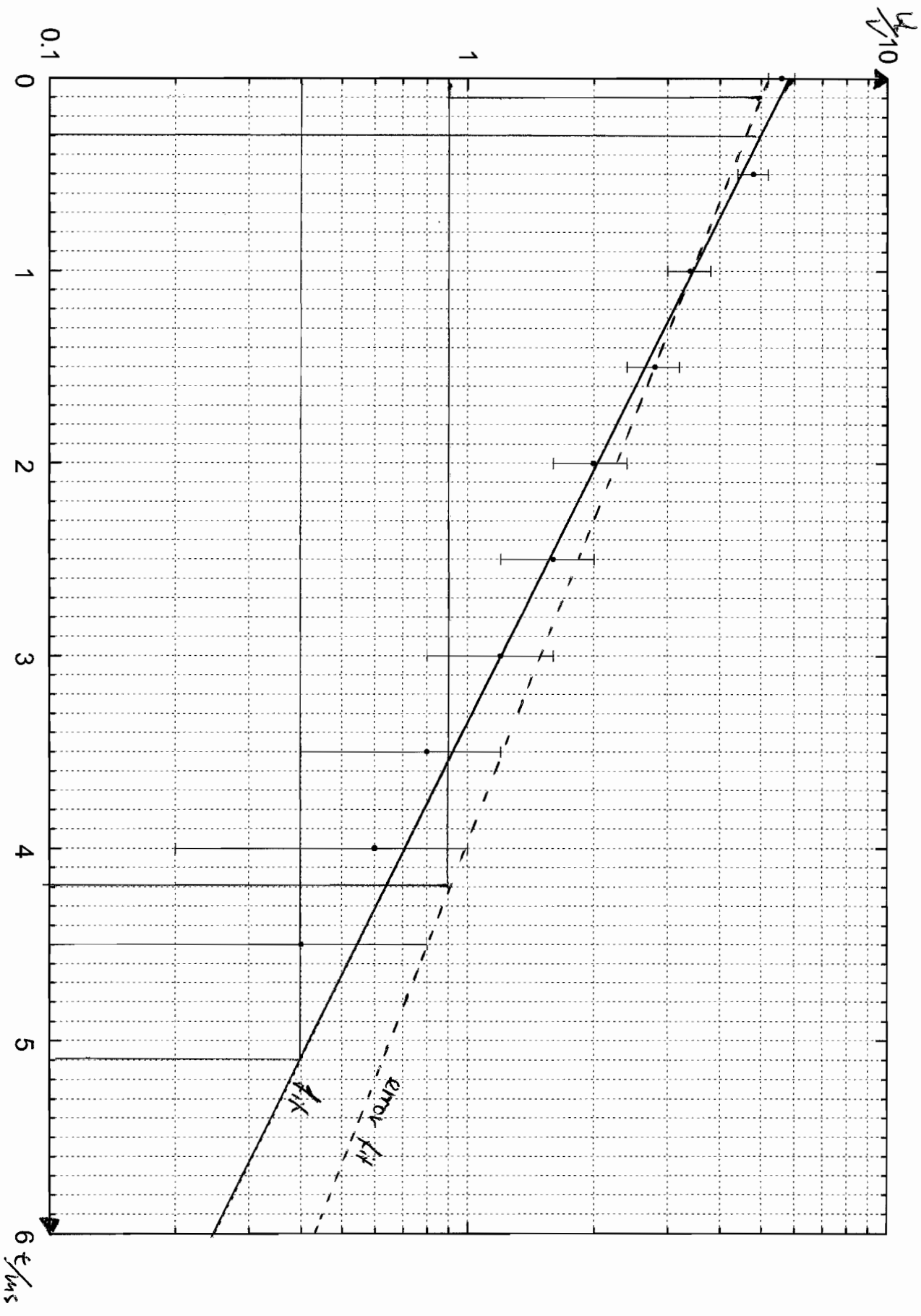
$$\bar{m} = ~~524,5~~ -(525 \pm 53) \frac{V}{s}$$

The slope of this plot is called the characteristic time  $\tau$ , which can be calculated as  $\tau = \frac{1}{RC}$ . From this follows:

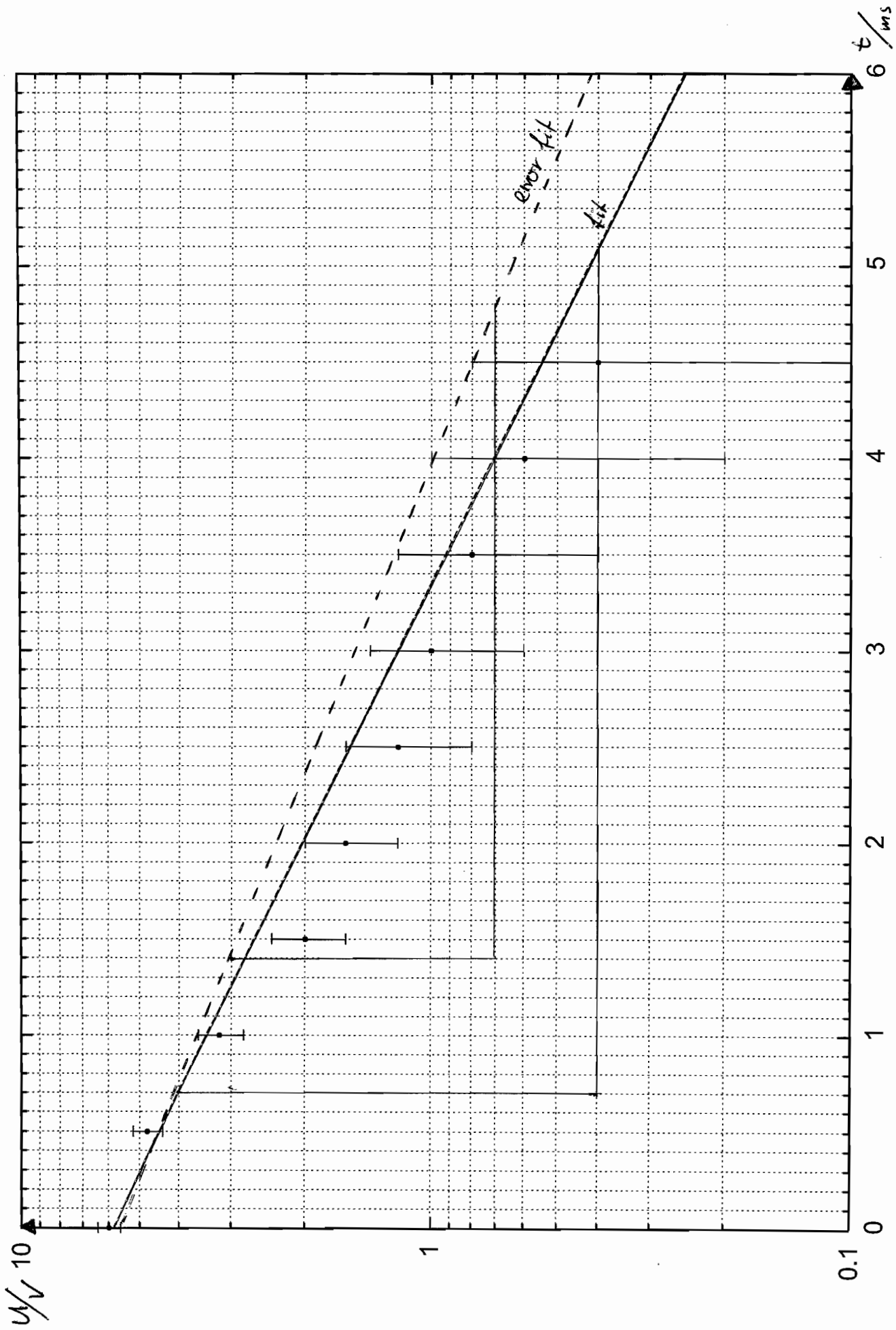
$$\tau = +(556 \pm 23) \frac{V}{s}$$

This values can be called equal, although the high error has to be discussed in the conclusion.

Measurement I Capacitor C<sub>1</sub>



Measurement II Capacitor C<sub>1</sub>



The second part of the experiment was a measurement with the capacitor  $C_2 = (1,007 \pm 0,010) \mu\text{F}$ . What we expect is a slower discharge which means that also  $\tau$  and the slope of the fit-plot is ~~then~~ expected to be much smaller.

This calculations refer to the plot on the right page:

Again, the slope can be calculated with the following result:

$$m_{C_2} = (67 \pm 17) \frac{\text{V}}{\text{s}}$$

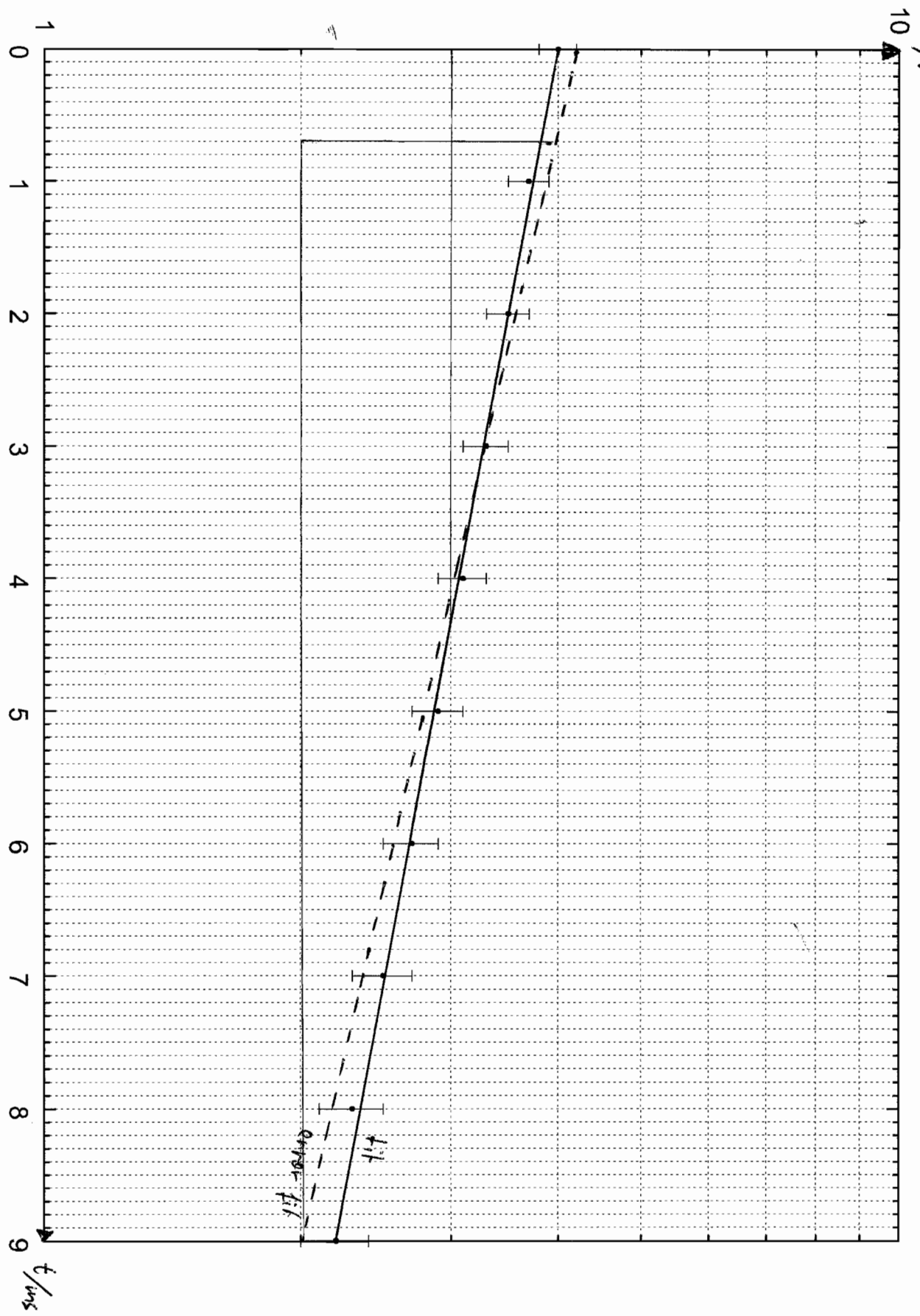
The characteristic time can be calculated as well:

$$\tau = (55 \pm 3) \frac{\text{V}}{\text{s}}$$

Again, this values can be called equal but only with a high error.

As a result of this experiment we can assess, that the time of discharge and therefore the characteristic time depends on the capacity of the capacitor. A higher capacity will cause a slower discharge. This brings the theory to proof in respect to the solution of the differential equation described into the introduction. The dependency on the resistor  $R$  has been tested in two undocumented measurements. In this case a higher resistance would cause a slower discharge as well. Of course, a ~~to~~ small resistor or capacitor would decrease time of discharge in the same way.

Measurement with capacitor  $C_2$



On Assignment 61:

The propose of this experiment was, to have a closer look on the dependency on frequency of the resistance of a capacitor. As described in the introduction, the absolute value is given as:

$$Z = \frac{1}{\omega C} = \frac{1}{2\pi f \cdot C}$$

$f$  being the frequency and  $C$  being the capacity of the measured capacitor. According to our measurement, we get the following table:

Frequency	Current	Error	Voltage	Error	Resistance (measured)	Resistance (calculated)
20	0.0375	0.0006	3.38	0.07	90133 ± 2359	7958 ± 398
50	0.0871	0.0010	3.29	0.07	37773 ± 914	3183 ± 96
100	0.1602	0.0020	3.19	0.07	19913 ± 503	1592 ± 32
200	0.800	0.010	3.22	0.07	4025 ± 101	796 ± 16
500	1.948	0.030	2.93	0.07	1504 ± 43	318 ± 3
1000	12.15	0.20	2.47	0.07	203 ± 7	159 ± 2
1500	21.1	0.5	2.55	0.07	121 ± 5	106 ± 1
2000	26.7	0.5	2.28	0.07	85 ± 4	80 ± 1
3013	35.9	0.6	1.79	0.06	50 ± 2	53 ± 1
4008	41.9	0.7	1.42	0.06	34 ± 2	40 ± 1
5010	45.6	0.7	1.14	0.06	25 ± 2	32 ± 1
6002	47.8	0.7	0.91	0.06	19 ± 2	27 ± 1
10072	48.9	0.7	0.31	0.06	6 ± 2	16 ± 1

The resistance labeled with "measured" results directly from our measurement of current and voltage using Ohm's law:

$$R = \frac{U}{I}$$

The "calculated" resistance results from the equation described above using the frequency and capacity.



By comparison, most of the values are significantly different. Only in the small range from 1500 Hz to 4000 Hz, we get compatible values.

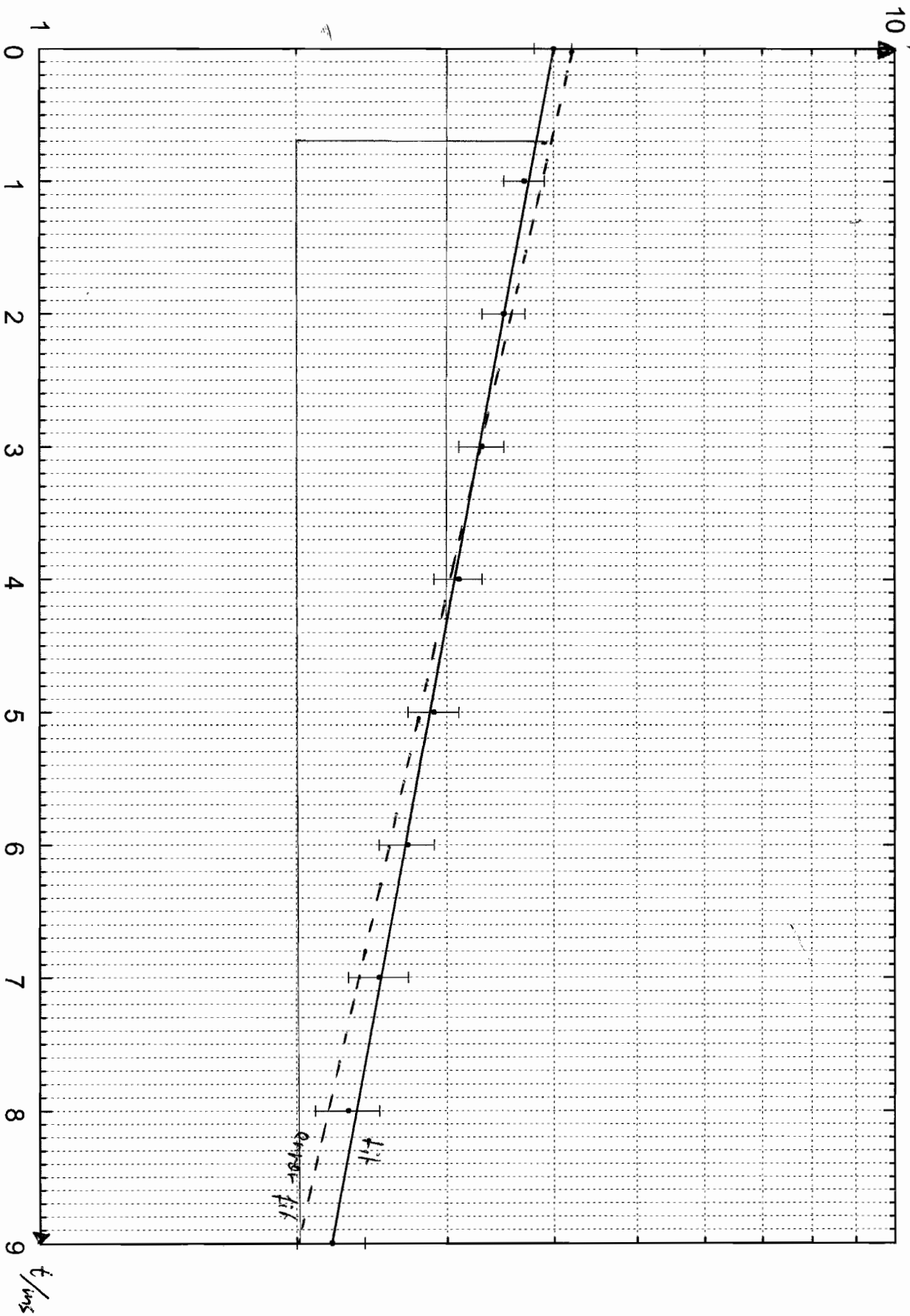
After a closer look on our experiment setup we detected, that the frequency generator was set to square-wave voltage, which was not intended. Therefore the values of current and voltage might not have been measured correctly.

However, the values follow a "one-over-x" function, as plotted on the right page. This, of course, was what we expected.

To get a more accurate proposition, a repetition of the whole experiment would be necessary.

The plot on the right page shows a limited range of measured values for the resistance in dependency of the frequency.

Measurement with capacitor  $C_2$



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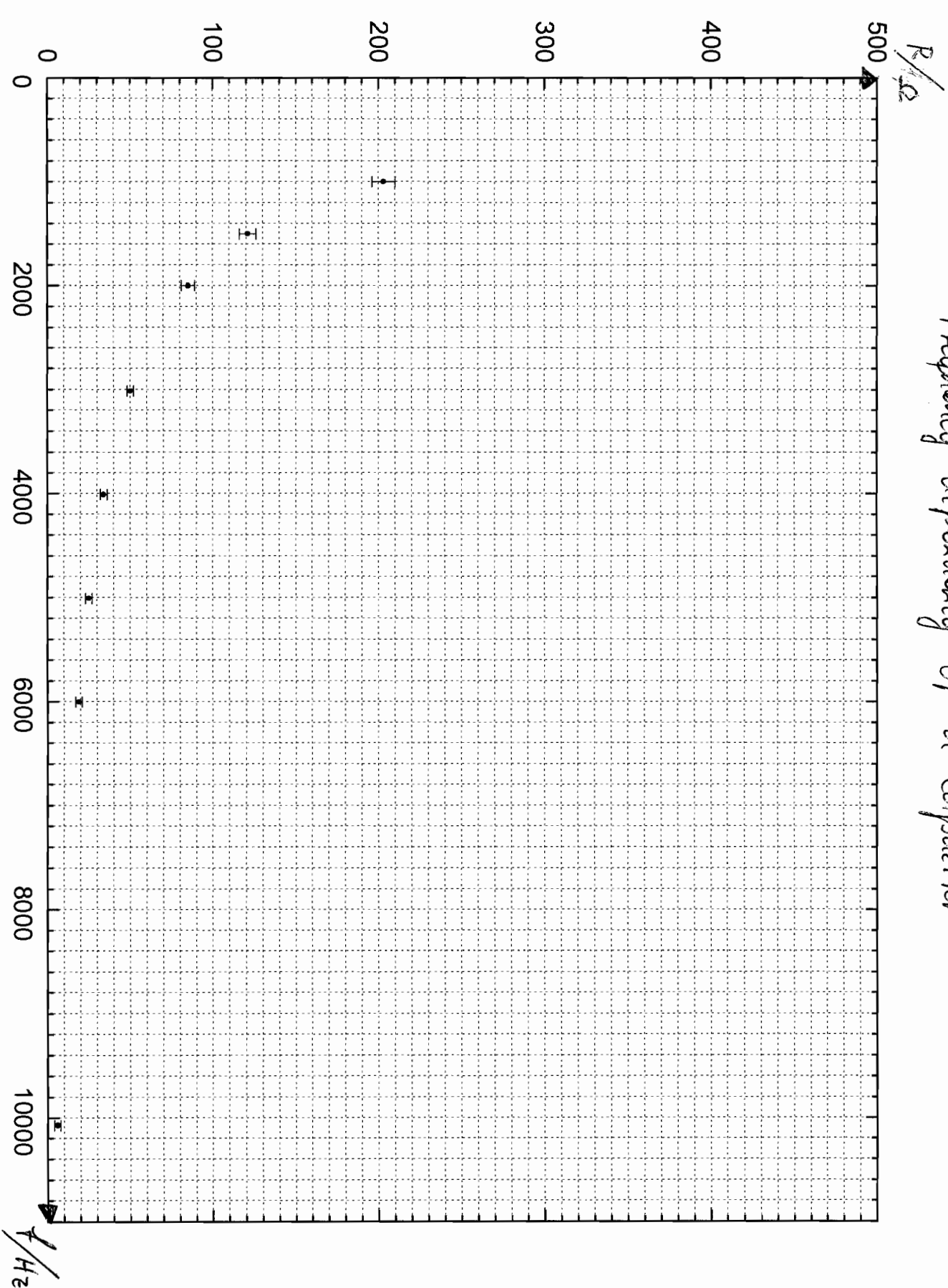
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Frequency dependency of a capacitor



## Conclusion

Most of the experiments have already been discussed during their analysis. Therefore, the main part of this conclusion will be about the applicability of the experiments for the specific tasks.

The purpose of experiment B1 was to determine the internal resistance of a power supply and a battery. Our expectation was a very small resistance, which will be hard to measure. Our apprehension was confirmed during the first measurement, which failed because of the high error of the external resistance. The second measurement resulted into values with an average error of 20 to 30 percent.

This means that systematic errors like the resistance of the connection wires have a high influence on the results. Finally, the experiment setup will not allow more accurate values than measured.

The Wheatstone bridge used in experiment C1 was ~~probably~~ probably the most accurate setup, ~~used~~ at all. The high difference between  $R_1$  and  $R_2$  allowed a very ~~precisions~~ ~~precisions~~ precise determination of the resistance of the copper wire. Therefore the value of the specific resistance calculated is very close to the literature value.

The high errors described during the analysis of experiment F1 have been caused by the insufficient reading accuracy of the oscilloscope. All other errors made have a minor importance. The usage of more precise devices would improve the accuracy more than any changes on the experiment setup.

Experiment G1 failed, because of a wrong experiment setup. The intention was to determine the AC-resistance of a capacitor using a sinus-voltage. In general this experiment should result into accurate values, because of the relatively high resistances expected. Systematic errors like the resistance of the wires, would just have a very small influence.