

# Implementations of OCT

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# OCT with Full Storage of Backward-Propagated States

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_t$  $\Psi_i$ 

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_{\text{bw}}(t)$  $\Psi_t$  $\Psi_i$ 

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

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$$\epsilon_1 \quad \epsilon_2 \quad \epsilon_{nt-2} \quad \epsilon_{nt-1}$$

$$\Psi_{\text{bw}}(t_0) \quad \Psi_{\text{bw}}(t) \quad \dots \quad \Psi_{\text{bw}}(t) \quad \Psi_t$$

$\Psi_i$

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

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# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\Psi_{\text{bw}}(t_0) \quad \Psi_{\text{bw}}(t) \quad \dots \quad \Psi_{\text{bw}}(t) \quad \Psi_t$$

$\epsilon_2 \quad \epsilon_{nt-2} \quad \epsilon_{nt-1}$

$$\Psi_i \quad \Psi_{\text{fw}}(t)$$

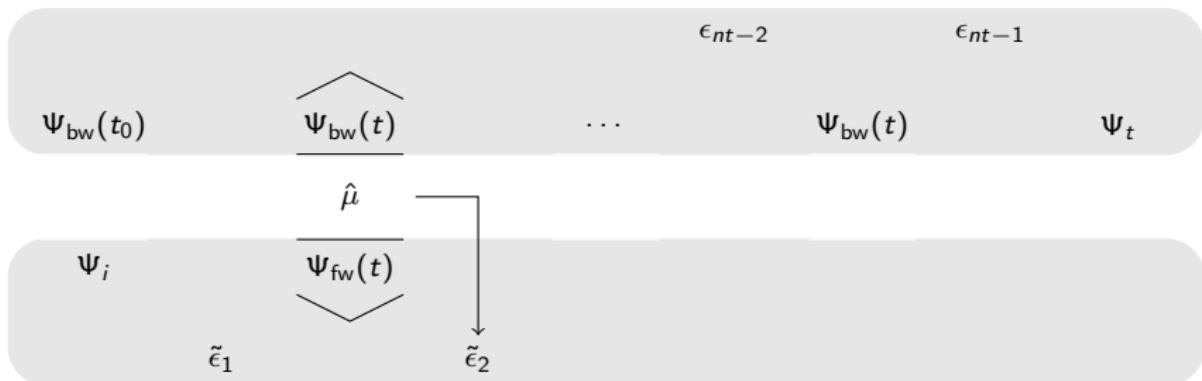
$\curvearrowright \tilde{\epsilon}_1 \curvearrowright$

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

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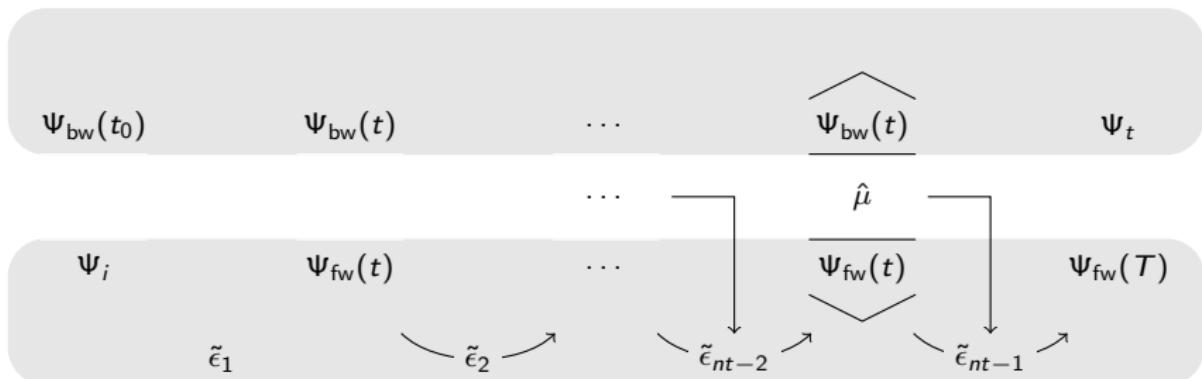


$$\begin{array}{cccccc} \times & & \times & & \times & \\ t_0 + \frac{dt}{2} & & t_0 + \frac{3}{2}dt & \cdots & T - \frac{3}{2}dt & T - \frac{dt}{2} \end{array}$$

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •



$$\begin{array}{cccccc} \times & & \times & & \times & \\ t_0 + \frac{dt}{2} & & t_0 + \frac{3}{2}dt & \dots & T - \frac{3}{2}dt & T - \frac{dt}{2} \end{array}$$

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\Psi_{\text{bw}}(t_0) \quad \Psi_{\text{bw}}(t) \quad \dots \quad \Psi_{\text{bw}}(t) \quad \Psi_t$$

$$\Psi_i \quad \quad \quad \quad \quad \Psi_{\text{fw}}(T)$$

$$\tilde{\epsilon}_1 \quad \tilde{\epsilon}_2 \quad \dots \quad \tilde{\epsilon}_{nt-2} \quad \tilde{\epsilon}_{nt-1}$$

$$\begin{matrix} \times & \times & \times & \times \\ t_0 + \frac{dt}{2} & t_0 + \frac{3}{2}dt & \dots & T - \frac{3}{2}dt \\ & & & T - \frac{dt}{2} \end{matrix}$$

# OCT with Full Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\Psi_{\text{bw}}(t_0)$$

$$\Psi_{\text{bw}}(t)$$

⋮

$$\Psi_{\text{bw}}(t)$$

$$\nearrow \\ \searrow \\ \Psi_t$$

$$= \tau$$

$$\Psi_i$$

$$\tilde{\epsilon}_1$$

$$\tilde{\epsilon}_2$$

$$\tilde{\epsilon}_{nt-2}$$

$$\tilde{\epsilon}_{nt-1}$$

$$\nearrow \\ \searrow \\ \Psi_{\text{fw}}(T)$$

$$\times \\ t_0 + \frac{dt}{2}$$

$$\times \\ t_0 + \frac{3}{2}dt$$

⋮

$$\times \\ T - \frac{3}{2}dt$$

$$\times \\ T - \frac{dt}{2}$$

# OCT with No Storage of Backward-Propagated States

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_t$  $\Psi_i$ 

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_{\text{bw}}(t)$  $\Psi_t$  $\Psi_i$ 

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

The diagram illustrates the temporal evolution of a state vector  $\Psi_{bw}$ . It shows a sequence of states  $\Psi_{bw}(t_0), \Psi_{bw}(t), \dots, \Psi_{bw}(t)$  arranged horizontally. Above the sequence, time points  $t_0, t = t_0 + dt, \dots, t = T - dt, T = t_0 + nt \cdot dt$  are listed. Brackets above the sequence indicate time steps  $\epsilon_1, \epsilon_2, \dots, \epsilon_{nt-1}$  corresponding to the intervals between consecutive states.

$$\Psi_{bw}(t_0) \quad \Psi_{bw}(t) \quad \dots \quad \Psi_{bw}(t) \quad \Psi_t$$

$\epsilon_1$        $\epsilon_2$        $\epsilon_{nt-2}$        $\epsilon_{nt-1}$

$\Psi_i$

The diagram shows the storage of state vectors  $\Psi_i$  at specific time points. The time points are marked with an 'x' above them:  $t_0 + \frac{dt}{2}, t_0 + \frac{3}{2}dt, \dots, T - \frac{3}{2}dt, T - \frac{dt}{2}$ .

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

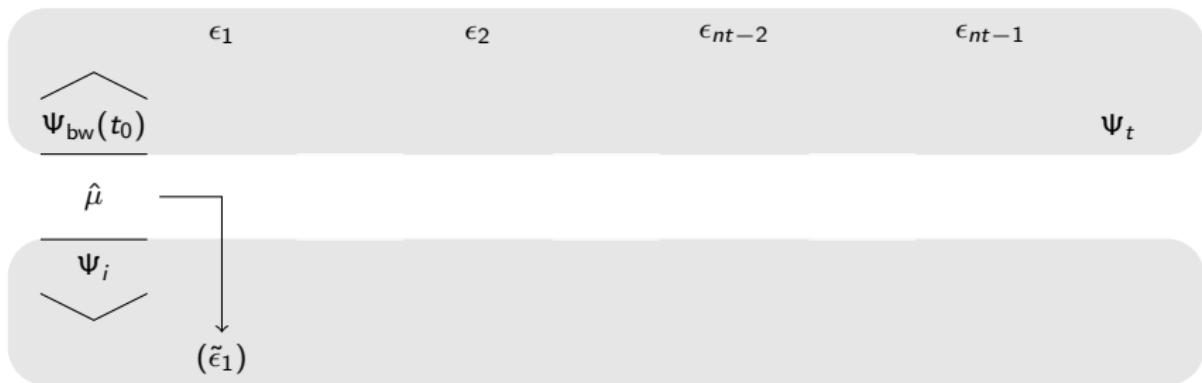
 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_{\text{bw}}(t_0)$  $\Psi_t$  $\Psi_i$ 

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •



$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$\epsilon_1$

$\Psi_{\text{bw}}(t_0)$

$\epsilon_2$

$\Psi_{\text{bw}}(t)$

$\epsilon_{nt-2}$

$\epsilon_{nt-1}$

$\Psi_t$

$\Psi_i$

$(\tilde{\epsilon}_1)$

$x$   
 $t_0 + \frac{dt}{2}$

$x$   
 $t_0 + \frac{3}{2}dt$

$\dots$

$x$   
 $T - \frac{3}{2}dt$

$x$   
 $T - \frac{dt}{2}$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

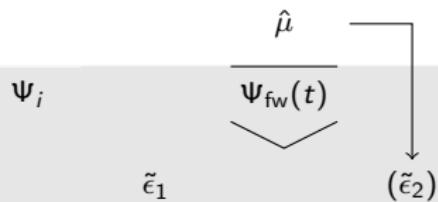
 $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_{\text{bw}}(t_0)$  $\Psi_{\text{bw}}(t)$  $\Psi_t$  $\Psi_i$  $\Psi_{\text{fw}}(t)$  $\curvearrowright \tilde{\epsilon}_1 \curvearrowright$  $x$   
 $t_0 + \frac{dt}{2}$  $x$   
 $t_0 + \frac{3}{2}dt$  $\dots$  $x$   
 $T - \frac{3}{2}dt$  $x$   
 $T - \frac{dt}{2}$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\overbrace{\Psi_{\text{bw}}(t)}^{\epsilon_2} \quad \overbrace{\Psi_{\text{fw}}(t)}^{\epsilon_{nt-2}} \quad \dots \quad \overbrace{\Psi_t}^{\epsilon_{nt-1}}$$



$$x \quad \quad \quad x \quad \quad \quad x \quad \quad \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\Psi_{\text{bw}}(t) \quad \dots \quad \overbrace{\Psi_{\text{bw}}(t)}^{\epsilon_{nt-2} \quad \epsilon_{nt-1}} \quad \Psi_t$$

$$\Psi_i \quad \Psi_{\text{fw}}(t) \quad \dots \quad \overbrace{\Psi_{\text{fw}}(t)}^{\hat{\mu}} \quad \Psi_{\text{fw}}(T)$$

$\tilde{\epsilon}_1 \quad \tilde{\epsilon}_2 \quad \dots \quad \tilde{\epsilon}_{nt-2} \quad \tilde{\epsilon}_{nt-1}$

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$\Psi_t$

$\Psi_i$

$\Psi_{\text{fw}}(T)$

$\tilde{\epsilon}_1$

$\tilde{\epsilon}_2$

$\tilde{\epsilon}_{nt-2}$

$\tilde{\epsilon}_{nt-1}$

$t_0 + \frac{dt}{2}$

$t_0 + \frac{3}{2}dt$

...

$T - \frac{3}{2}dt$

$T - \frac{dt}{2}$

# OCT without Storage

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\begin{array}{c} \diagup \\ \Psi_t \\ \diagdown \end{array} = \tau$$

$\Psi_i$

$\tilde{\epsilon}_1$

$\tilde{\epsilon}_2$

$\tilde{\epsilon}_{nt-2}$

$\tilde{\epsilon}_{nt-1}$

$$\begin{array}{c} \Psi_{fw}(T) \\ \diagup \\ \diagdown \end{array}$$

$$t_0 + \frac{dt}{2}$$

$$t_0 + \frac{3}{2}dt$$

...

$$T - \frac{3}{2}dt$$

$$T - \frac{dt}{2}$$

# OCT with Segmented Storage of Backward-Propagated States

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_t$  $\Psi_i$ 

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_{\text{bw}}(t)$  $\Psi_t$  $\Psi_i$ 

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\epsilon_1 \quad \epsilon_2 \quad \epsilon_{nt-2} \quad \epsilon_{nt-1}$$

$$\Psi_{\text{bw}}(t_0) \quad \Psi_{\text{bw}}(t) \quad \dots \quad \Psi_{\text{bw}}(t) \quad \Psi_t$$

$\Psi_i$

$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

 $\epsilon_1$  $\epsilon_2$  $\epsilon_{nt-2}$  $\epsilon_{nt-1}$  $\Psi_{\text{bw}}(t_0)$  $\Psi_{\text{bw}}(t)$  $\Psi_t$  $\Psi_i$ 

$$\times \\ t_0 + \frac{dt}{2}$$

$$\times \\ t_0 + \frac{3}{2}dt$$

 $\dots$ 

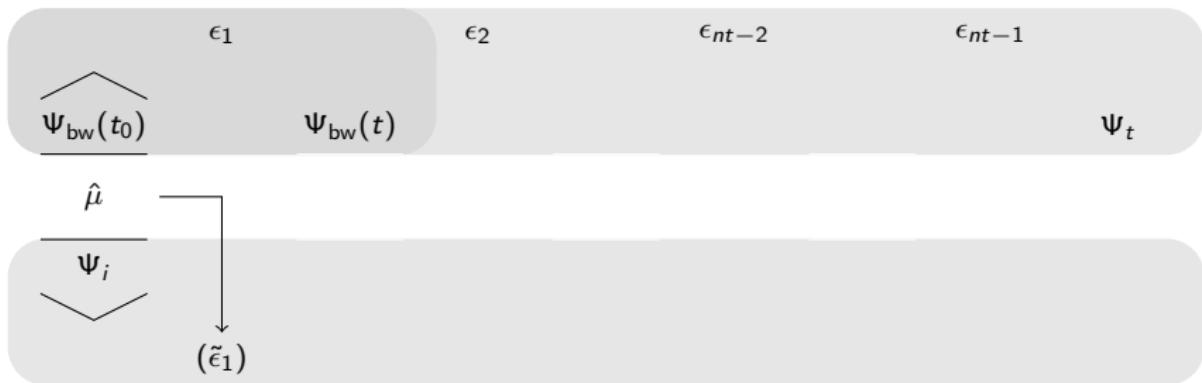
$$\times \\ T - \frac{3}{2}dt$$

$$\times \\ T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

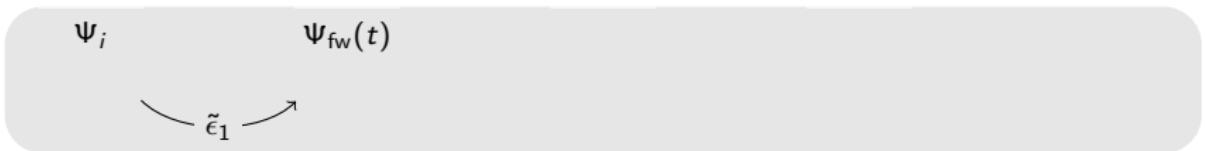
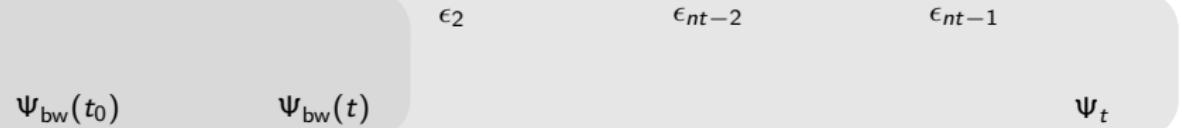


$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

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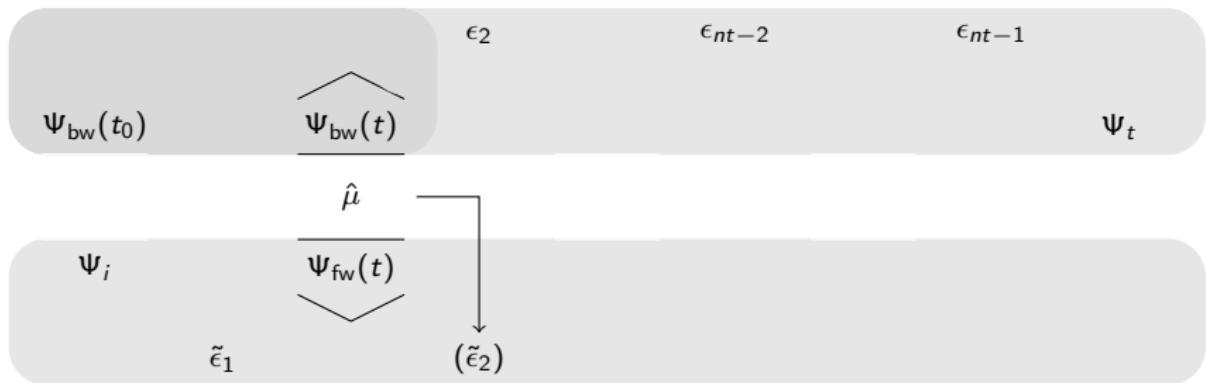


$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •



$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\epsilon_2$$

$$\epsilon_{nt-2}$$

$$\epsilon_{nt-1}$$

$$\Psi_{\text{bw}}(t)$$

$$\Psi_{\text{bw}}(t)$$

$$\Psi_t$$

$$\Psi_i$$

$$\Psi_{\text{fw}}(t)$$

$$\tilde{\epsilon}_1$$

$$(\tilde{\epsilon}_2)$$

$$\times \\ t_0 + \frac{dt}{2}$$

$$\times \\ t_0 + \frac{3}{2}dt$$

...

$$\times \\ T - \frac{3}{2}dt$$

$$\times \\ T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\epsilon_2 \quad \quad \quad \quad \quad \epsilon_{nt-2} \quad \quad \quad \quad \quad \epsilon_{nt-1}$$


$$\Psi_{\text{bw}}(t) \quad \quad \quad \Psi_{\text{bw}}(t) \quad \quad \quad \Psi_{\text{bw}}(t) \quad \quad \quad \Psi_t$$

$$\Psi_i \quad \quad \quad \Psi_{\text{fw}}(t)$$

$$\tilde{\epsilon}_1 \quad \quad \quad (\tilde{\epsilon}_2)$$

$$\times \quad \quad \quad \times \quad \quad \quad \times \quad \quad \quad \times$$
$$t_0 + \frac{dt}{2} \quad \quad \quad t_0 + \frac{3}{2}dt \quad \quad \cdots \quad \quad T - \frac{3}{2}dt \quad \quad \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\epsilon_{nt-2} \quad \epsilon_{nt-1}$$

$$\Psi_{\text{bw}}(t) \quad \Psi_{\text{bw}}(t) \quad \Psi_{\text{bw}}(t) \quad \Psi_t$$

$$\Psi_i \quad \Psi_{\text{fw}}(t) \quad \Psi_{\text{fw}}(t)$$

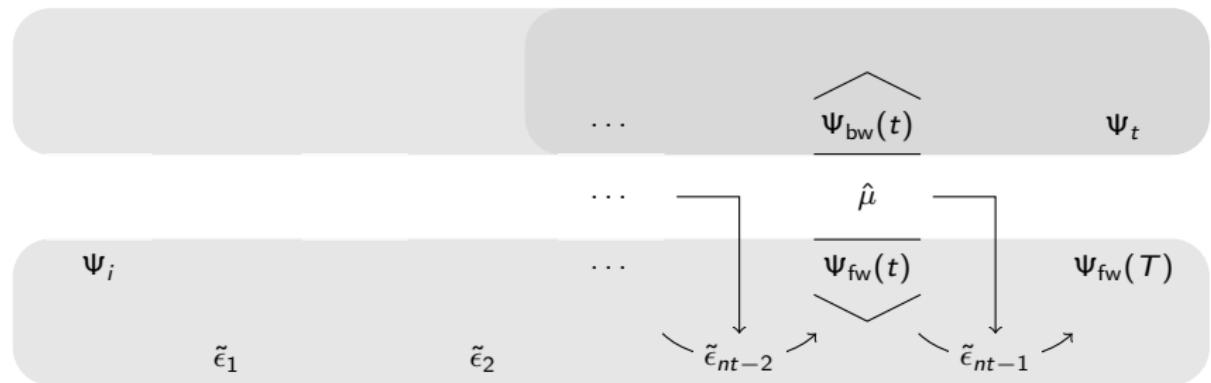
$$\tilde{\epsilon}_1 \quad \curvearrowright \quad \tilde{\epsilon}_2 \quad \nearrow$$

$$\begin{matrix} \times & \times & \times & \times \\ t_0 + \frac{dt}{2} & t_0 + \frac{3}{2}dt & \dots & T - \frac{3}{2}dt \\ & & & T - \frac{dt}{2} \end{matrix}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •



$$x \quad x \quad x \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\dots \quad \Psi_{\text{bw}}(t) \quad \Psi_t$$

$$\Psi_i \quad \quad \quad \Psi_{\text{fw}}(T)$$

$$\tilde{\epsilon}_1 \quad \tilde{\epsilon}_2 \quad \dots \quad \tilde{\epsilon}_{nt-2} \quad \tilde{\epsilon}_{nt-1}$$

$$x \quad \quad \quad x \quad \quad \quad x \quad \quad \quad x$$
$$t_0 + \frac{dt}{2} \quad t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt \quad T - \frac{dt}{2}$$

# OCT with Segmentation

$$t_0 \quad t = t_0 + dt \quad \dots \quad t = T - dt \quad T = t_0 + nt \cdot dt$$

•                    •                    •                    •

$$\begin{array}{c} \diagup \\ \Psi_t \\ \diagdown \end{array} = \tau$$

$\Psi_i$

$\tilde{\epsilon}_1$

$\tilde{\epsilon}_2$

$\tilde{\epsilon}_{nt-2}$

$\tilde{\epsilon}_{nt-1}$

$$\begin{array}{c} \Psi_{fw}(T) \\ \diagup \\ \diagdown \end{array}$$

$$x \\ t_0 + \frac{dt}{2}$$

$$x \\ t_0 + \frac{3}{2}dt \quad \dots \quad T - \frac{3}{2}dt$$

$$x \\ T - \frac{dt}{2}$$