Optimal Control of Transmon Qubit Gates in the Presence of Decoherence

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Abstract

We consider two transmon qubits\textsuperscript{[1]} coupled via a cavity bus\textsuperscript{[2]}. The strong coupling of each qubit to the shared cavity modes provides an indirect interaction that in addition to the direct qubit-qubit interaction can be used to implement a two-qubit gate (e.g., CPHASE). Describing the system numerically allows us to take into account an arbitrary number of qubit and cavity excitations. Going beyond the dispersive limit permits the implementation of fast gates, which are necessary to beat decoherence. Optimal control theory (OCT), specifically Krotov’s method\textsuperscript{[3]}, is used to find microwave pulses that drive the full system in the desired way in the shortest possible amount of time. The complete system Hamiltonian for complex dynamics with OCT can fully exploit. We show results from such an optimization on a CPHASE gate, for different pulse durations. We also discuss decoherence and analyze the influence of spontaneous decay of the cavity on the gate fidelity, and give an outlook on how OCT may find robust pathways.

\section{Two Transmon Qubits Coupled via Cavity Bus}

superconducting qubits inside a transmission line resonator. Fig. from [5]

\begin{equation}
\hat{H} = -\frac{\hbar}{2} \sum_j (\hat{b}_j^\dagger \hat{b}_j + \hat{b}_j \hat{b}_j^\dagger) + \sum_{j,k} \frac{J_{jk}}{2} \hat{b}_j^\dagger \hat{b}_k + \sum_{j,k} \frac{J_{jk}}{2} \hat{b}_k^\dagger \hat{b}_j
\end{equation}

\section{Optimization: Krotov Method}

We optimize for \( \delta \rightarrow \text{CPHASE} \) by minimizing the functional \( J \) containing the gate fidelity \( F \) and a running cost ensuring monostability: i.e., a scaling parameter \( \lambda_F \) and a shape function \( S(t) \).

\begin{equation}
J(\phi_k(t)) = -F(\phi_k(T)) + \lambda_F \int_0^T S(t) \frac{dF}{dt}(\phi_k(t)) dt
\end{equation}

with \( \Delta_\phi = e^{-\delta(t)} - e^{\delta(t)} \) for \( \phi_k(t) \in \{00\}, \{01\}, \{10\}, \{11\} \).

Note: Direct qubit-qubit coupling is weak; entanglement is primarily reached indirectly via interaction with the cavity \( \phi \).

\section{Decoherence}

Open system dynamics with a master equation in Lindblad form [4].

\begin{equation}
\frac{d\rho}{dt} = -i[\rho, H] + \sum_{\alpha \gamma} \gamma_{\alpha \gamma} D(\gamma_{\alpha \gamma}) \rho + 2\gamma_{\alpha \gamma} D(\gamma_{\alpha \gamma})^\dagger \rho + \sum_{\alpha} \gamma_{\alpha \gamma} D(\gamma_{\alpha \gamma})^\dagger \rho + \sum_{\alpha} \gamma_{\alpha \gamma} D(\gamma_{\alpha \gamma})^\dagger \rho + \sum_{\alpha} \gamma_{\alpha \gamma} D(\gamma_{\alpha \gamma})^\dagger \rho
\end{equation}

The parameters \( \gamma_{\alpha \gamma} \) are decay, dephasing and leakage rates, respectively. "Cavity decay" is described by \( \gamma_{00} \).

It is straightforward to write the Krotov update equation (3) for Liouvillian space, using density matrices instead of states and using Eq. (4) for propagation. It can be shown that it is sufficient to use only three density matrices as "basis states":

\begin{equation}
\rho_1 = \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right), \quad \rho_2 = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right), \quad \rho_3 = \left( \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right)
\end{equation}

Possible choices for a fidelity: \( F = \text{tr} \left[ (\hat{\rho}_{00}(t) - \hat{\rho}_{11}(t))^2 \right] \). Any other distance measure on density matrices may be used.

References


\section{Optimal Control & Outro}

Optimal control successfully finds fast gates at fidelities at the quantum error correction threshold. Gates are sufficiently fast to almost beat decoherence.

The optimization of a CPHASE gate illustrates the extremely rich dynamics that the Hamiltonian Eq. (1) provides: Two other qubit quantum gates are possible as well, and may be more robust in the presence of decoherence.

Pulses may proliferate a significant number of higher qubit and cavity states, requiring a description beyond an effective two-qubit model. Optimizing from appropriate guess pulses, the number of qubit and cavity levels can be kept reasonably low (e.g., 4.5-level pulses, 20 cavity levels).

Optimize in Liouvillian space, with full decoherence model (Liouvillian equation), improving the fidelity under discount - possible to the quantum error correction threshold. This can be done efficiently using the approach presented in Section III. The choice of fidelity may have a significant effect on the optimization success and should be explored systematically.

Ultimately, use the local invariant function [6] in Liouvillian space to optimize for the two-qubit gate least susceptible to decoherence.