Optimal Controlled Phasegates for Trapped Neutral Atoms at the Quantum Speed Limit

Michael Goerz*
Tommaso Calarco†
Christiane P. Koch*

*Universität Kassel
†Universität Ulm

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Universal Quantum Computing

**Motivation**

**Theoretical Model and Optimization Method**

Two Calcium Atoms at Short Internuclear Distance

Two Atoms at Long Distance under Strong Dipole-Dipole Interaction

**Universal Quantum Computing**

**Controlled Phasegate**

\[
\hat{O}(\chi) = \text{CPHASE}(\chi) = \begin{pmatrix}
e^{i\chi} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

**Controlled-Not**

\[\text{CNOT} = \]

- CPHASE(\(\pi\)) equivalent to CNOT \(\Rightarrow\) Universal Quantum Computing
- CPHASE is used in Quantum Fourier Transform

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Optimal Controlled Phasegates for Trapped Neutral Atoms
Two-Qubit Gates on Trapped Neutral Atoms

Calcium:

- $^1S_0 \rightarrow |0\rangle$
- $^1P_1 \rightarrow |a\rangle$
- $^1P_3 \rightarrow |1\rangle$

$\omega_L = 23652 \text{ cm}^{-1}$

- Low-Lying states in Alkaline-Earth atoms or Rydberg states
- Atoms in optical lattice or optical tweezers
The Objective

Problem

- QC with atomic collisions: adiabaticity $\Rightarrow$ slow.
- Strong interaction $\Rightarrow$ fast gates?
  - only if ignoring motion.

Quantum Speed limit

- QSL: What is the maximum speed at which a quantum system can evolve?
- What limits on the gate duration can we find through optimization?
- How do gate durations depend on the interaction strength?
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Approach

- Describe the system including the motional degree of freedom.
- Optimize for varying times / interaction strengths:
  - Two Calcium atoms at fixed distance (fixed interaction): vary $T$
  - For fixed $T$, two atoms with “artificial” dipole-dipole interaction $V(R) = -C_3/R^3$: vary $C_3$
Theoretical Model and Optimization Method

Two-Qubit-Hamiltonian, Optimization with Krotov
System Hamiltonian

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System Hamiltonian

\[ d_{x1} \rightarrow \text{integrate out COM} \]

\[ R_0 = d \]

\[
\begin{align*}
47304.61 \text{ cm}^{-1} & \quad |aa\rangle \\
38862.37 \text{ cm}^{-1} & \quad |a1\rangle \\
30420.13 \text{ cm}^{-1} & \quad |0a\rangle \\
23652.30 \text{ cm}^{-1} & \quad |a0\rangle \\
15210.06 \text{ cm}^{-1} & \quad |1a\rangle \\
0.0 \text{ cm}^{-1} & \quad |00\rangle \\
\end{align*}
\]
Optimizing the Laser Pulse

Target Functional

\[ J = -\frac{1}{N} \text{Re} \left[ \text{tr} \left( \hat{O}^\dagger \hat{U} \right) \right] + \int_0^T \frac{\alpha}{S(t)} \Delta \epsilon^2(t) \, dt; \quad \hat{O} = \text{CPHASE} \]
\[ \hat{U} = e^{-i\hat{H}(\epsilon(t))t} \]

Krotov: pulse update \( \Delta \epsilon \) minimizing \( J \)

\[ \Delta \epsilon \sim \text{Im} \left\langle \Psi_{bw} | \hat{\mu} | \Psi_{fw} \right\rangle \]

Palao, Kosloff,
PRA 68, 062308 (2003)
## Measures of Merit

*Fidelity $F$ and cost functional $J$ are not very informative.*

### Control over the Motional Degree of Freedom

$$F_{00} = \left| \langle 00(R) | \hat{U}(T, 0; \epsilon^{opt}) | 00(R) \rangle \right|^2$$

Does $|00\rangle$ return to its initial **vibrational eigenstate**?

### Gate Phases

$$\phi_{00} = \arg \left( \langle 00(R) | \hat{U}(T, 0; \epsilon^{opt}) | 00(R) \rangle \right)$$

What is the **phase change** relative to the initial state?

### True Two-Qubit Phase

- Cartan Decomposition leads to
  $$\chi = \phi_{00} - \phi_{01} - \phi_{10} + \phi_{11}$$
- Concurrence (Entanglement)
  $$C = \left| \sin \frac{\chi}{2} \right|$$
Two Calcium Atoms at Short Internuclear Distance

For which gate durations can we reach a high-fidelity CPHASE?
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Two Atoms at Long Distance under Strong Dipole-Dipole Interaction

Parameters of the Optimization

- Short internuclear distance
  ⇒ sufficient interaction

- Peak intensity $\epsilon_0$
  to induce 1 Rabi cycle

- Pulse duration between $T_{\text{int}}^{1\text{rad}} = 1.23$ ps and $T_v = 800$ ps

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Optimization Success over Pulse Duration

⇒ For small $T$, vibrational purity is lost with increasing two-qubit phase

⇒ High two-qubit phase \textit{and} high vibrational only for long pulse durations

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Optimal Controlled Phasegates for Trapped Neutral Atoms
System Dynamics for 800 ps Pulse

\[ F = 0.997 \]

\[ \tau_{00} = \left\langle 00(R) \right| \hat{U}(T, 0; \epsilon^{opt}) \left| 00(R) \right\rangle \]
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Can we avoid vibration with very short pulses, but very strong interaction?
Parameters of the Optimization

- Fixed short pulse duration
  \( T = 1 \text{ ps}, \ T = 0.5 \text{ ps} \)

- Realistic lattice spacing
  with strong interaction \( \sim -\frac{C_3}{R^3} \)

- Vary \( C_3 \):
  - \( C_3 = 1 \times 10^6 \)
    Action over 1 ps for Calcium at \( d = 5 \text{ nm} \), scaled to \( d = 200 \text{ nm} \)
  - Increase by three orders of magnitude
    Action over 800 ps for Calcium at \( d = 5 \text{ nm} \), scaled to \( d = 200 \text{ nm} \)

\[ C_3 = 1 \times 10^6 \]
\[ \vdots \]
\[ C_3 = 1 \times 10^9 \]
Optimization Success over Dipole Interaction Strength

- Increasing two-qubit-phase with increasing interaction strength
- For small $T$, vibrational purity is lost with increasing two-qubit phase
Conclusions
Conclusions

- Long gate duration can reach arbitrarily high fidelities.
- For short gate durations, the two-qubit phase is at the expense of the vibrational purity.
- If $T < QSL$, not all measures of merit can be fulfilled.
- Time scale for a successful gate is determined by $\max(T_{int}, T_{vib})$. 
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