Q56.45 Construction of a Fast Two-Qubit Gate for Ultracold Atoms Using Optimal Control

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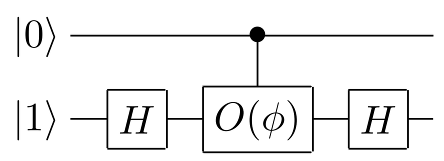
Introduction

In recent years, a number of physical implementations of Quantum Computing have been examined, such as cavity QED, trapped ions, NMR, or SQUID-systems. We consider an alternative model based on neutral ultracold atoms in an optical lattice [1]. The qubits can be encoded in the electronic or hyperfine levels of the atoms. An appropriately shaped laser pulse couples to the electronic states and drives arbitrary quantum-computational operations. Single qubit operations are easy to achieve. We have implemented a numerical scheme to find laser pulses that perform a two-qubit phasegate. Our goal consists in calculating short, high fidelity pulses for the realization of this target gate.

Universal Quantum Computing

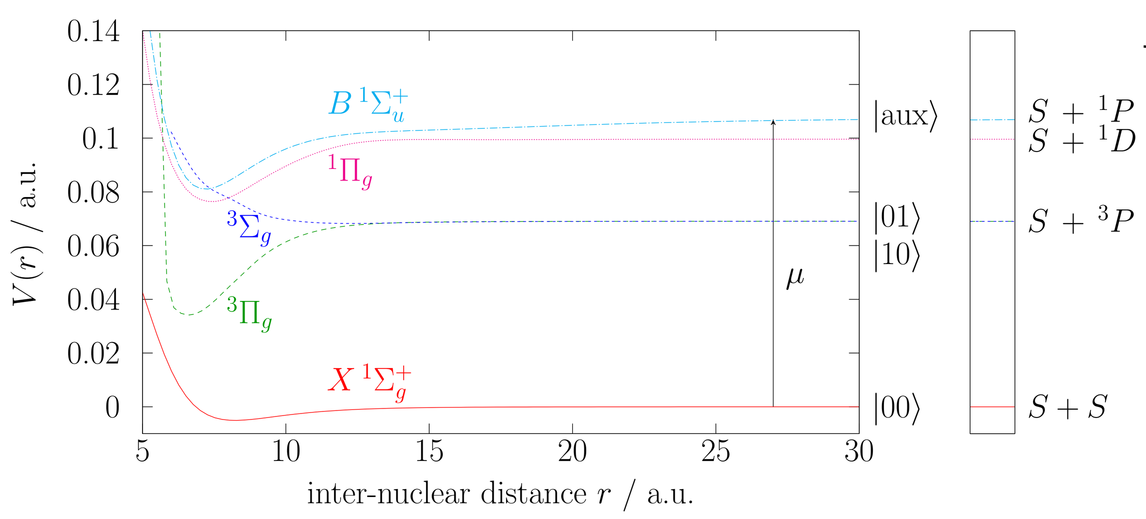
The set of all one-qubit gates plus the two-qubit CNOT is universal. More generally, the CNOT is equivalent to the controlled phasegate, combined with two Hadamard gates.

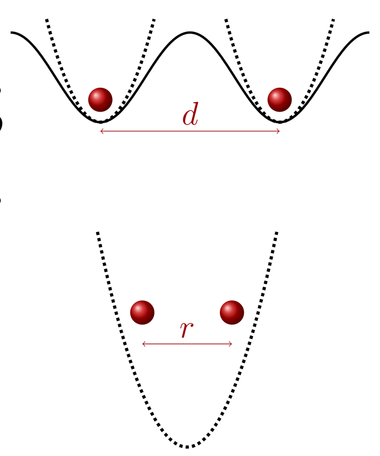
$$\hat{O}(\phi) = \begin{pmatrix} e^{i\phi} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$\text{CNOT} \rightarrow \hat{O}(\pi) \qquad \sqrt{\text{SWAP}} \rightarrow \hat{O}(\pi/2)$$

Qubit Encoding and Gate for Ca2 System





- single qubits: ${}^1S_0$ state is $|0\rangle$, ${}^3P_1$ state is $|1\rangle$
- two atoms in harmonic trap potential; relative coordinates, integrate out COM
- For $r < \infty$: Born-Oppenheimer molecular potentials
- two qubit basis (electronic surfaces): $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. $X^1\Sigma_g^+$ surface is $|00\rangle$
- laser pulse drives transition between $|00\rangle$ and $B^1\Sigma_u^+$ $|\text{aux}\rangle$ surface

$$\hat{H}_{2q} = \begin{pmatrix} \hat{T} + \hat{V}_{00}(r) + \hat{V}_{\text{trap}}(r,d) & \hat{\mu_{21}}(r)\,\epsilon(t) \\ \hat{\mu_{12}}(r)\,\epsilon(t) & \hat{T} + \hat{V}_{\text{aux}}(r) + \hat{V}_{\text{trap}}(r,d) \end{pmatrix}$$

- goal: change phase of only the $|00\rangle$ eigenstate.

$$\Psi_{\text{rel}}(r) \approx \left(\frac{\mu\omega_0}{4\pi\hbar}\right)^{1/4} \sum_{\pm} e^{\frac{-m\omega_0}{2\hbar}(d\pm r)^2} \qquad \text{trap groundstate}$$

$$\Psi_{\pm}(x,t) \longrightarrow \Psi_{\pm}(x) e^{i\phi_{\pm}(x,t)} \qquad \text{time evolution in absolute coordinate system}$$

$$\Psi_{00}(r,t) = \Psi_{\text{rel}}(r) \otimes |0\rangle\,|0\rangle$$
$$\downarrow$$
$$\Psi'_{00}(r,t) = e^{-i(\phi_+(r,t)+\phi_-(r,t))}\,\Psi_{\text{rel}}(r) \otimes |0\rangle\,|0\rangle = e^{-i\chi_{00}}\,\Psi_{\text{rel}}(r) \otimes |00\rangle$$

Finding an Optimal Pulse

Starting from a guess pulse, an optimal pulse implementing the target operation $\hat{O}$ can be found by minimization of the target functional J [2, 3]:

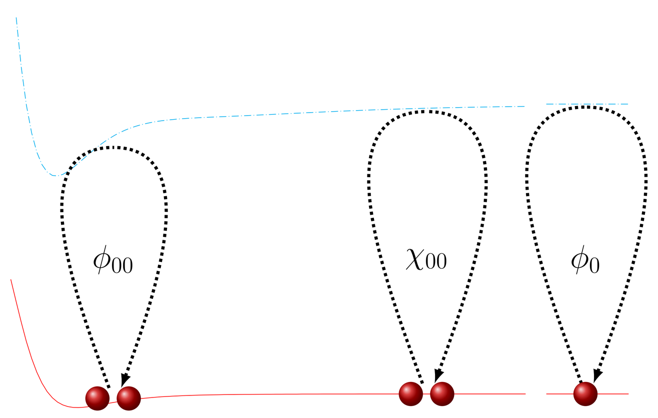
$$J = -F_{re} + \int_0^T \frac{\alpha}{S(t)} (\Delta\epsilon(t))^2\,; \qquad F_{re} = \frac{1}{N}\,\text{Re}\left[\sum_{l=1}^{N} \left\langle l \left| \hat{O}^\dagger \hat{U}(T;0;\epsilon) \right| l \right\rangle\right]; \qquad \text{OCT} \Rightarrow \Delta\epsilon$$

- $|l\rangle$ are the N initial states of the system
- $\hat{O}\,|l\rangle$ are target states, $\hat{U}(T;0;\epsilon)\,|l\rangle$ are states propagated from $t=0$ to $t=T$ with the pulse $\epsilon(t)$.
- phase sensitive fidelity F_{re} is calculated from the overlap between the target states and the propagated states
- second part of J is constraint of the time evolution: field changes should converge within the pulse time; pulse shape $S(t)$ enforces smooth switching on/off. α is a multiplier strengthening the constraint.

The Optimal Control Theory (OCT) algorithm finds a modification $\Delta\epsilon(t)$ to the guess pulse $\epsilon(t)$ that is guaranteed to decrease J.

One-Qubit and Two-Qubit Phases

- driving the interacting system always affects the non-interacting system as well; but we want a ***true*** two-qubit operation.



$$\hat{H}_{1q} = \begin{pmatrix} E_0 & \hat{\mu}\,\epsilon(t) \\ \hat{\mu}\,\epsilon(t) & E_{\text{aux}} \end{pmatrix}$$

$$\chi_{00}(r,t) = \phi_{00}(r,t) + \phi_0(t)$$

χ_{00} : phase from system evolution

ϕ_{00} : true interaction phase $\stackrel{!}{=} \pi$ (CNOT)

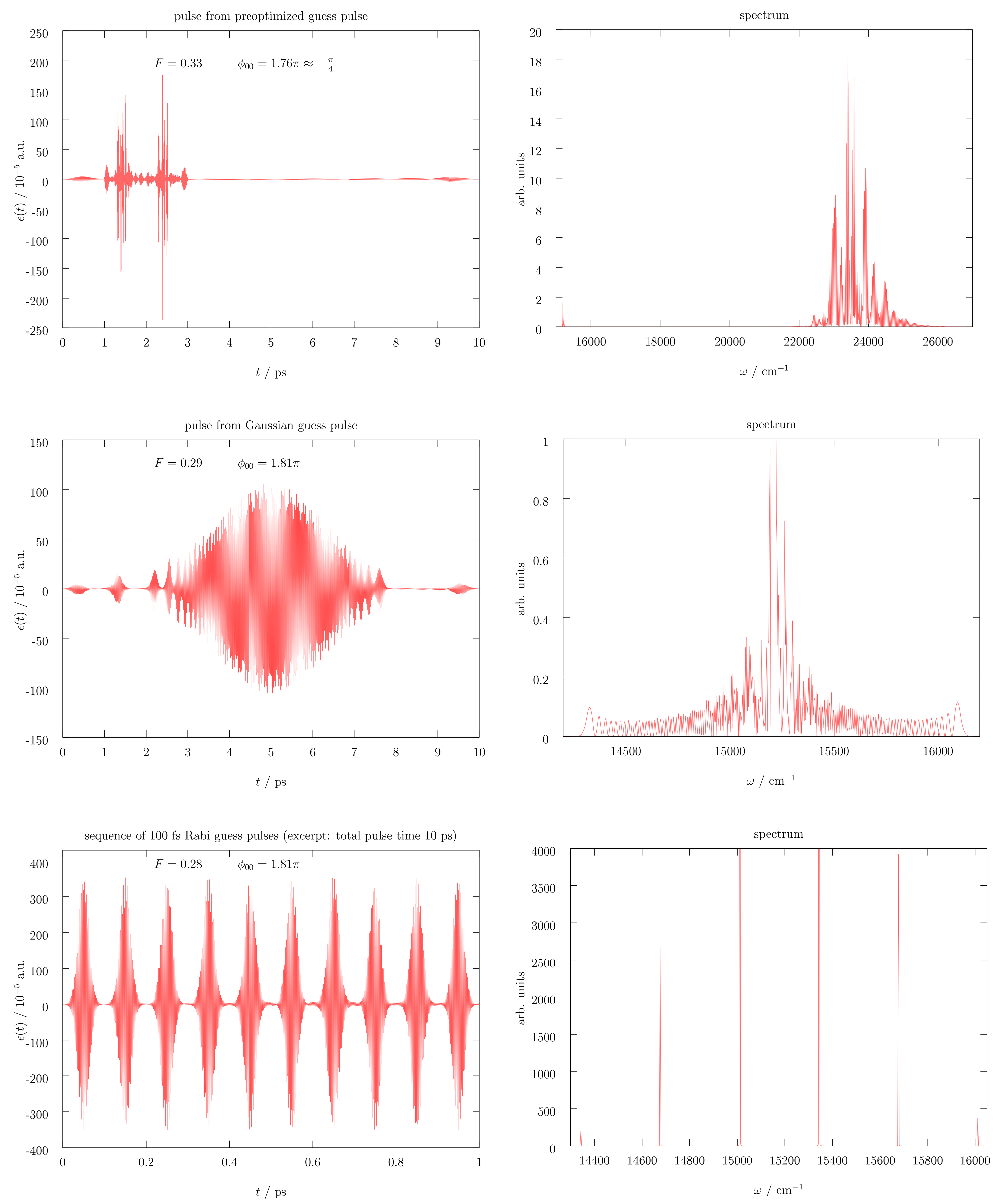
ϕ_0 : non-interacting phase

- optimize both the interacting and the non-interacting system in parallel with a single pulse (two state-to-state transitions)
- target for CNOT is $\chi_{00} \stackrel{!}{=} \pi$; $\phi_0 \stackrel{!}{=} 0$. This implies that the true two-qubit-phase fulfills the target condition.
- condition is too strict: only $\chi_{00} - \phi_0 = \pi$ is required

System Parameters and Search Strategies

- trap distance d: only large values are experimentally feasible. Current calculations are at $d = 10\,\text{nm}$, ultimate goal is $d = 75\,\text{nm}$ [4]
- pulse time: larger values for d require more time for the pulse. For $d = 10\,\text{nm}$, $T = 10\,\text{ps}$
- pulse intensity: more population transfer
- multi-photon transitions: use interference to make pulse "dark" for non-interacting system
- increase α: allow more changes to intensity
- use informed guess pulses, e.g. based on Franck-Condon factors.

Optimized Pulses



Outlook

- find pulses for better fidelities and experimentally more feasible parameters
- accumulate target phase by repeating a pulse sequence
- formulate OCT functionals directly in terms of ϕ_{00}: loosen constraint on on-qubit-phase
- apply the method to Rb system: better known system, easy to work with for experimentalists; but: more complicated, qubit encoding in hyperfine levels.

References

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[4] A. V. Gorshkov et. al. *Phys. Rev. Lett.* **100**, 093005 (2008).