Role of Dissipation for Optimal Control of Rydberg Gates

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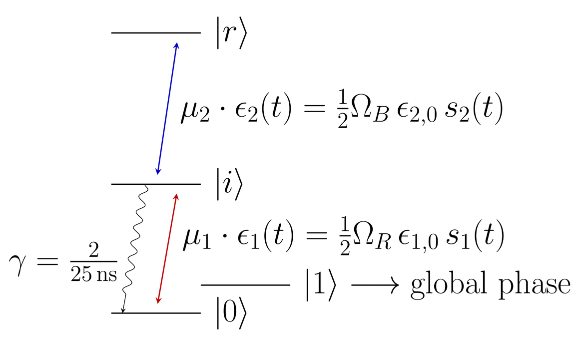
Abstract

Optimal control theory can find optimal pulses to implement a two-qubit controlled phasegate at the quantum speed limit [1]. Here, we investigate optimal control strategies to realize a two-qubit Rydberg gate in the presence of dissipation. Two rubidium atoms are trapped in optical tweezers, with the qubits encoded in the hyperfine levels of the electronic ground state. The Rydberg state $|r\rangle$ is accessed via an intermediate state $|i\rangle$. Spontaneous emission from $|i\rangle$ and $|r\rangle$ as well as fluctuations of the Rydberg state might lead to decoherence. The dynamics are described by a master equation of Lindblad form. We test several variants of the Krotov algorithm on the dissipative system, and optimize the Rydberg gate under spontaneous emission from $|i\rangle$, for varying dissipation rates. Our preliminary results demonstrate the viability of our optimization approach, and yield results corresponding to known strategies for avoiding dissipation.

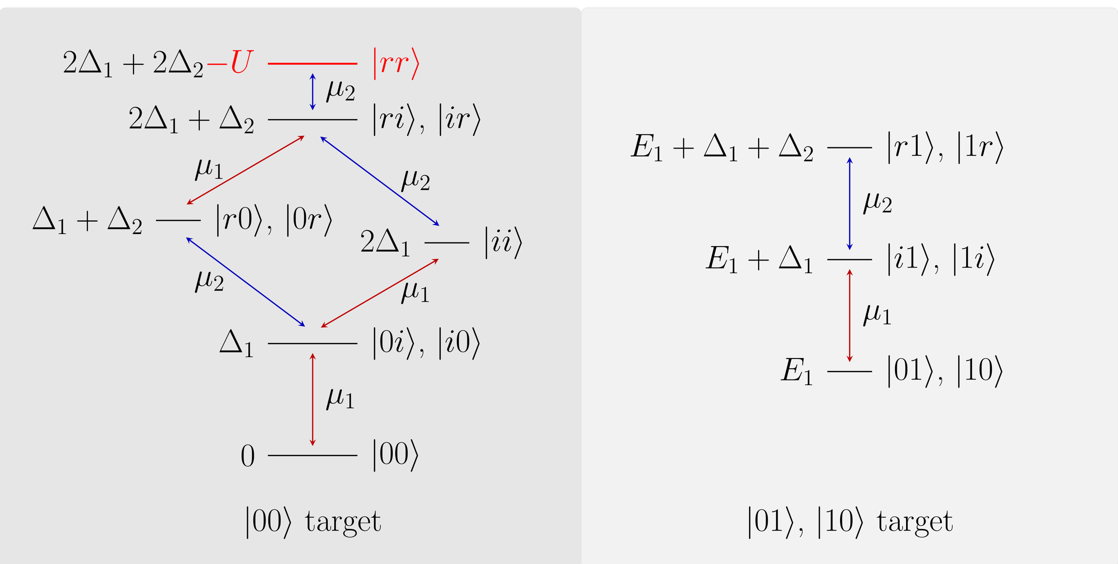
① Qubit Encoding in the Rydberg System

In RWA:
$$\hat{\mathbf{H}}_{1q}(t) = \begin{pmatrix} 0 & 0 & \mu\epsilon_1(t) & 0 \\ 0 & E_1 & 0 & 0 \\ \mu\epsilon_1(t) & 0 & \Delta_1 & \mu\epsilon_2(t) \\ 0 & & \mu\epsilon_2(t) & \Delta_1+\Delta_2 \end{pmatrix}$$

with detunings Δ_1 for $|0\rangle \to |i\rangle$, Δ_2 for $|i\rangle \to |r\rangle$



$\hat{\mathbf{H}}_{2q} = \hat{\mathbf{H}}_{1q} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbf{H}}_{1q} - U\,|rr\rangle\langle rr| =$



System Parameters [2]

- $\Omega_R = 25 \cdot 2\pi$ MHz
- $\Omega_B = 300 \cdot 2\pi$ MHz
- $E_1 = 6.8$ GHz
- $\Delta_1 = 600$ MHz
- $\Delta_2 = 0$
- $U = 50$ MHz

Pulse Parameters

- $T = 41$ ns
- $\epsilon_{1,0} = 100$, $\epsilon_{2,0} = 10$

② Optimization Algorithm: Krotov Method

Minimize $J[\{\phi_k\}, \epsilon] = -F[\{\phi_k(T)\}] + \int_0^T g_a(\epsilon, t)\,\mathrm{d}t + \int_0^T g_b(\{\phi_k\}, t)\,\mathrm{d}t$

Update formula [3] for $\Delta\epsilon = \epsilon^{(1)} - \epsilon^{(0)}$,

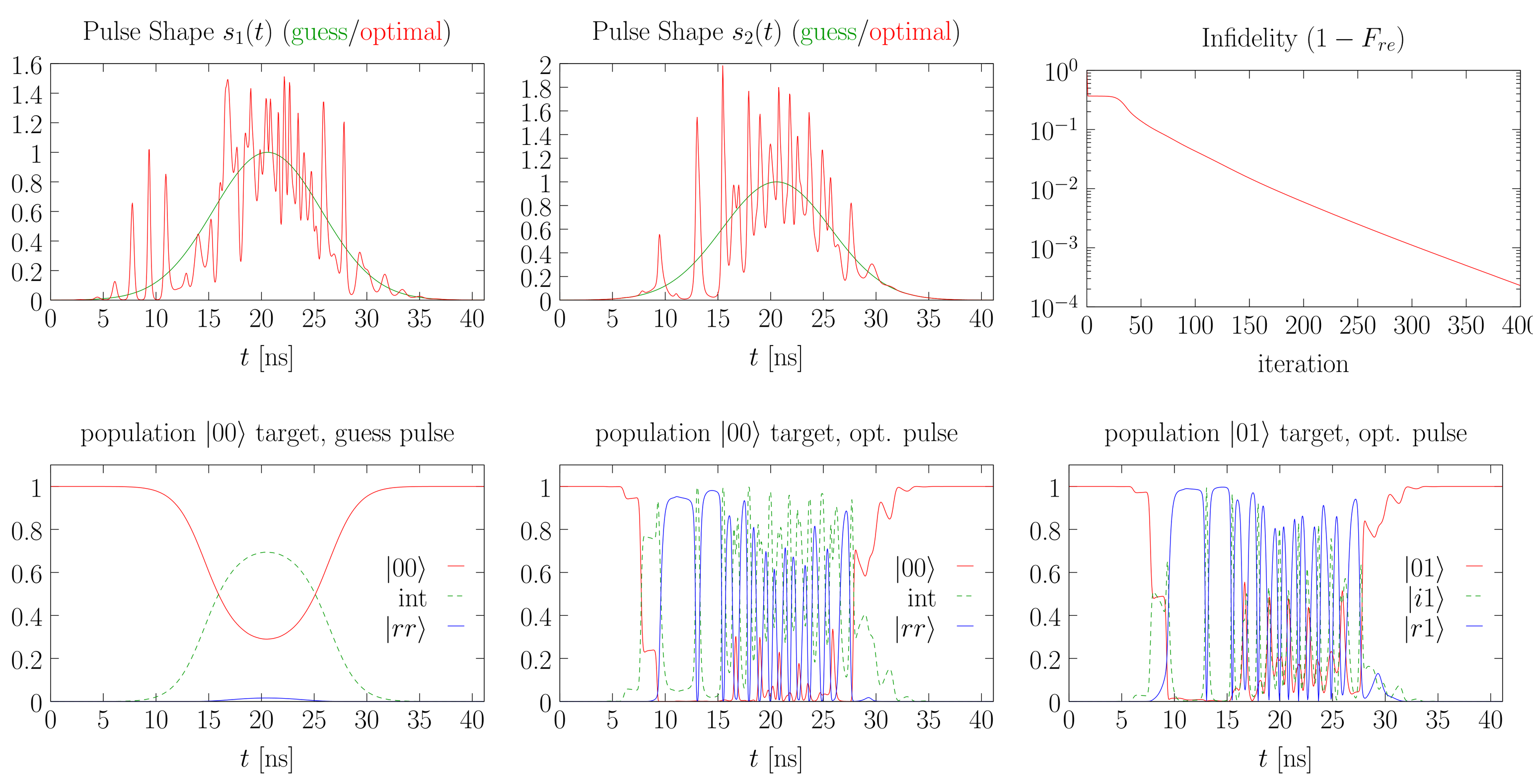
$$\Delta\epsilon = \frac{S(t)}{\lambda_a} \mathfrak{Im} \left\{ \sum_k \left\langle \chi_k^{(0)}(t) \middle| \left(\frac{\partial \hat{\mathbf{H}}}{\partial \epsilon}\right)_{\epsilon^{(1)}} \middle| \phi_k^{(1)}(t) \right\rangle + \frac{1}{2}\sigma(t) \sum_k \left\langle \Delta\phi_k(t) \middle| \left(\frac{\partial \hat{\mathbf{H}}}{\partial \epsilon}\right)_{\epsilon^{(1)}} \middle| \phi_k^{(1)}(t) \right\rangle \right\}, \quad (1)$$

where $|\phi_k\rangle(t) = \hat{\mathbf{U}}\,|\phi_k(0)\rangle$ are the forward propagated states and the $|\chi_k(t)\rangle$ are the backward-propagated states, with the boundary condition $|\chi_k(T)\rangle = \frac{\partial F}{\partial \langle \phi_k|}$ given by the final time functional F, e.g. with $\left|\phi_k^{\mathrm{tgt}}\right\rangle = \hat{\mathbf{O}}\,|\phi_k(0)\rangle$ [4],

$$F_{\mathrm{re}} = \frac{1}{N} \mathfrak{Re} \sum_{k=1}^{N} \langle \phi_k^{\mathrm{tgt}} | \phi_k(t) \rangle \quad \Longrightarrow \quad |\chi_k\rangle = \frac{1}{N}\frac{1}{2} \left|\phi_k^{\mathrm{tgt}}\right\rangle,$$

$$F_{\mathrm{sm}} = \frac{1}{N^2} \left| \sum_{k=1}^{N} \langle \phi_k^{\mathrm{tgt}} | \phi_k(t) \rangle \right|^2 \quad \Longrightarrow \quad |\chi_k\rangle = \frac{1}{N^2} \sum_{k'=1}^{N} \langle \phi_{k'}^{\mathrm{tgt}} | \phi_{k'} \rangle \left|\phi_k^{\mathrm{tgt}}\right\rangle.$$

③ Optimization Results in Hilbert Space



References

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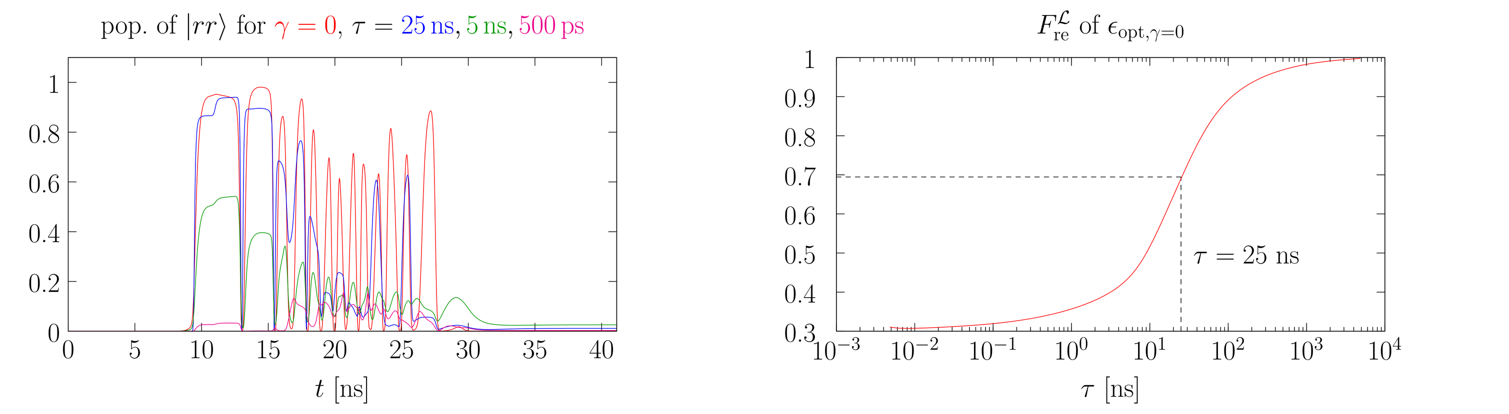
④ Effect of Decoherence

System dynamics are modeled by master equation of Lindblad form, with dissipation rates γ_k and Lindblad operators $\hat{\mathbf{A}}_k$,

$$\frac{\partial}{\partial t}\hat{\rho} = -i\hat{\mathcal{L}}\hat{\rho} = -i\left[\hat{\mathbf{H}}, \hat{\rho}\right] + \sum_k \gamma_k \left(\hat{\mathbf{A}}_k \hat{\rho} \hat{\mathbf{A}}_k^\dagger - \frac{1}{2}\left[\hat{\mathbf{A}}_k^\dagger \hat{\mathbf{A}}_k, \hat{\rho}\right]_+\right).$$

We only consider relaxation from $|i\rangle \to |0\rangle$, described by $\hat{\mathbf{A}} = |0\rangle\langle i|$ and $\gamma = \frac{2}{\tau}$, $\tau = 25\,$ns. In the two-qubit system, this becomes $\hat{\mathbf{A}}_{2q} = \hat{\mathbf{A}} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbf{A}}$.



⑤ Optimization in Liouville Space

Unitary target $\hat{\mathbf{O}}$ now implies optimization of $\hat{\rho}_k \to \hat{\rho}_k^{\mathrm{tgt}} = \hat{\mathbf{O}}\hat{\rho}_k\hat{\mathbf{O}}^\dagger$ for full set of Liouville basis states $\{\hat{\rho}_k\}$, with the Krotov update formula for $\Delta\epsilon = \epsilon^{(1)} - \epsilon^{(0)}$,

$$\Delta\epsilon(t) = \frac{S(t)}{\lambda_a} \mathfrak{Im} \left\{ \left\langle\!\left\langle \hat{\Xi}_k^{(1)} \middle| \left(\frac{\partial \hat{\mathcal{L}}}{\partial \epsilon}\right)_{\epsilon^{(1)}} \middle| \rho_k^{(1)} \right\rangle\!\right\rangle + \frac{1}{2}\sigma(t) \sum_k \left\langle\!\left\langle \Delta\rho_k(t) \middle| \left(\frac{\partial \hat{\mathcal{L}}}{\partial \epsilon}\right)_{\epsilon^{(1)}} \middle| \rho_k^{(1)}(t) \right\rangle\!\right\rangle \right\},$$

where the $\hat{\Xi}_k$ need to be chosen specifically for the given final time functional. Two approaches:

- Formulate Liouville equivalents of Hilbert space functionals [5], e.g.:

$$F_{\mathrm{re}}^{\mathcal{L}} = \frac{1}{N}\mathfrak{Re} \sum_{k=1}^{N} \left\langle\!\left\langle \rho_k^{\mathrm{tgt}} \middle| \rho_k(t) \right\rangle\!\right\rangle, \qquad F_{\mathrm{sm}}^{\mathcal{L}} = \frac{1}{N^2} \left| \sum_{k=1}^{N} \left\langle\!\left\langle \rho_k^{\mathrm{tgt}} \middle| \rho_k(t) \right\rangle\!\right\rangle \right|^2 \quad \Rightarrow \quad \hat{\Xi}_k^{\mathcal{L}} = \frac{\partial \hat{\mathcal{L}}}{\partial \langle\!\langle \rho |}$$

  Can't be done for functionals depending only indirectly on the states but directly on $\hat{\mathbf{U}}$, e.g. the gate fidelity $F_U = \frac{1}{N}\mathfrak{Re}\,\mathrm{Tr}\left[\hat{\mathbf{O}}^\dagger \hat{\mathbf{U}}\right]$ or the local invariants functional [6]

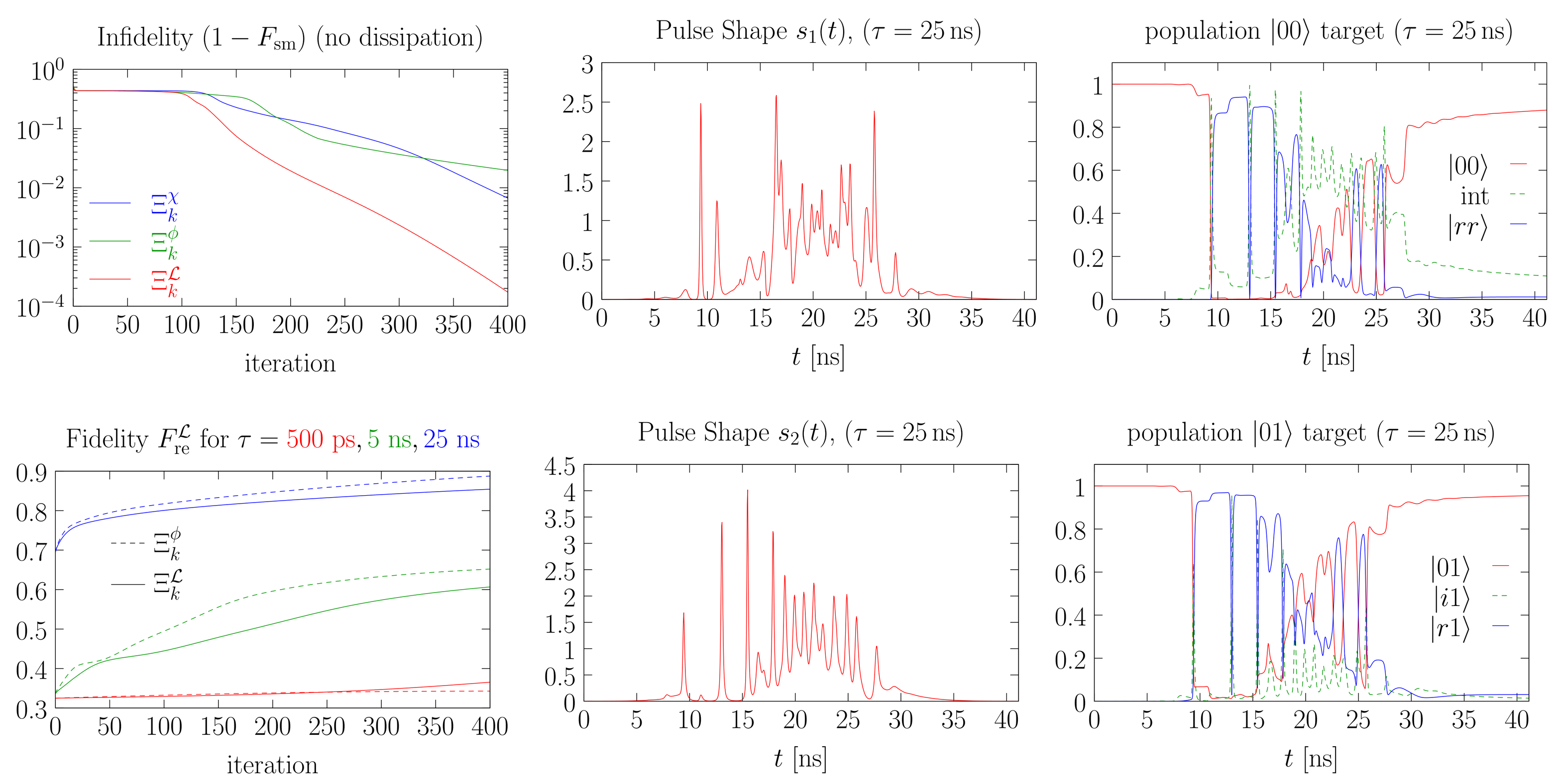
$$F_{\mathrm{LI}} = \Delta g_1^2(\hat{\mathbf{U}}) + \Delta g_2^2(\hat{\mathbf{U}}) + \Delta g_3^2(\hat{\mathbf{U}}) - \frac{1}{N}\mathrm{Tr}\left[\hat{\mathbf{U}}\hat{\mathbf{U}}^\dagger\right]; \qquad g_1, g_2, g_3 = \text{local invariants}$$

- Construct $\hat{\Xi}_{(ij)}$ from $\{|\phi_k\rangle, |\chi_k\rangle\}$ in Eq. 1.
  - Intuitively, from just $\{|\chi_k\rangle\}$: $\hat{\Xi}_{(ij)}^{\chi} = |\chi_i\rangle\langle\chi_j|$
  - From combination of $\{|\chi_k\rangle, |\phi_k\rangle\}$: $\hat{\Xi}_{(ij)}^{\phi} = \frac{1}{2}|\phi_i(T)\rangle\langle\chi_j| + \frac{1}{2}|\chi_i\rangle\langle\phi_j(T)|$; Motivation:

$$\frac{\mathrm{d}F}{\mathrm{d}\hat{\rho}_{nm}} = \sum_k \left[\frac{\mathrm{d}F}{\mathrm{d}\,|\phi_k\rangle} \cdot \frac{\mathrm{d}\,|\phi_k\rangle}{\mathrm{d}\,|\phi_n\rangle\langle\phi_m|} + \frac{\mathrm{d}F}{\mathrm{d}\,\langle\phi_k|} \cdot \frac{\mathrm{d}\,\langle\phi_k|}{\mathrm{d}\,|\phi_n\rangle\langle\phi_m|} \right]; \qquad \text{ansatz for} \quad \frac{\mathrm{d}\,|\phi_k\rangle}{\mathrm{d}\,|\phi_n\rangle\langle\phi_m|}$$

  - Other possibilities? ⇒ **How can $\hat{\mathbf{U}}$ be recovered from dissipative dynamics?**

⑥ Optimization Results in the Dissipative System



Comparison of $\hat{\Xi}_k^{\mathcal{L}}$, $\hat{\Xi}_k^{\chi}$, $\hat{\Xi}_k^{\phi}$ for optimization with $F_{\mathrm{sm}}^{\mathcal{L}}$:

The $\hat{\Xi}_k^{\mathcal{L}}$ directly derived in Liouville space generally outperforms $\hat{\Xi}_k^{\chi}$ and $\hat{\Xi}_k^{\phi}$, but both approaches are viable. However, convergence for choices of $\hat{\Xi}_k$ constructed from Hilbert space states cannot be guaranteed mathematically, especially in the case of non-unitary evolution.

Optimization under dissipation was done with the optimal pulses in the closed system as guess pulses.

The optimization has limited success in counteracting the dissipation. The optimized pulses tend toward bang-bang type, which are limited in their numerical representability, due to the finite time grid.

We expect that for longer pulse durations, adiabatic dynamics – the other known solution [6, 7] to avoiding dissipation – could be found.

⑦ Conclusions & Outlook

We have formulated three approaches to use the Krotov algorithm for gate optimization on a dissipative system: Either the use of fidelities defined directly in Liouville space with an associated $\hat{\Xi}_k^{\mathcal{L}}$, or the construction of $\hat{\Xi}_k$ from the states of the closed Hilbert space, in two variants.

For gate optimization, the definition of the functional is only rigorous if the unitary evolution associated with an evolution in Liouville space can be extracted. In general, it remains an open question how to do this.

A continuing investigation of the optimization of the Rydberg gate under dissipation will show whether optimal control will to be able to find non-trivial solutions that go beyond either bang-bang or adiabatic behavior.