Qubit Encoding in the Rydberg System

In RWA, \( \hat{H}_0(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & E_1(t) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_2(t) \end{pmatrix} \), with detunings \( \Delta_2 \) for \( |0⟩ \rightarrow |1⟩ \), \( \Delta_2 \) for \( |0⟩ \rightarrow |3⟩ \),
\[
\hat{H}_{\text{int}} = \hat{H}_0 + \Delta - \frac{\mu}{2} (|1⟩⟨1| + |3⟩⟨3|)
\]
with \( \gamma \) the Rydberg gate under spontaneous emission from e.g. with \( \Delta \), 
\[
\hat{H} = \hat{H}_0 + \mu \sigma_z \frac{\sigma_z}{2}
\]
where \( \sigma_z \) is the Pauli matrix.

System Parameters:
- \( \Delta_0 = 25 \) GHz
- \( \Delta_1 = 300 \) MHz
- \( E_1 = 6.8 \) GHz
- \( \Lambda = 600 \) MHz
- \( \gamma = 100 \) MHz

Pulse Parameters:
- \( T = 41 \) ns
- \( \epsilon_{1,0} = 100 \), \( \epsilon_{2,1} = 10 \)

Optimization Algorithm: Krotov Method

Minimum \( F(\phi_1, \phi_2) = -\int F_\text{int}(\phi_2(T)) + \int F_\text{ext}(\phi_1, \phi_2) \text{d}t \)

Update formula \([3]\) for \( \Delta \):
\[
\Delta = \frac{\delta S}{\delta \Delta} \bigg|_{\psi_{\text{free}}} = \frac{\delta}{\delta \Delta} \int \left( \sum_{k=1,2} \langle \phi_k(t) | \hat{H}_0 | \phi_k(t) \rangle + \frac{1}{2} \sum_{k=1,2} \langle \phi_k(t) | \hat{H}_1 | \phi_k(t) \rangle + \frac{1}{2} \sum_{k=1,2} \langle \phi_k(t) | \hat{H}_2 | \phi_k(t) \rangle \right) \text{d}t.
\]

Optimization Results in Hilbert Space

\[ \hat{H}_{\text{int}} = \hat{H}_0 + \mu \sigma_z \frac{\sigma_z}{2} \]

\[ \hat{H} = \hat{H}_0 + \mu \sigma_z \frac{\sigma_z}{2} \]

References


Effect of Decoherence

System dynamics are modeled by master equation of Lindblad form, with dissipation rates \( \gamma_1 \) and \( \gamma_2 \) of the Lindblad operators \( \hat{A}_1 \) and \( \hat{A}_2 \):
\[
\frac{\partial}{\partial t} |\Psi(t)⟩ = -i [\hat{H}, |\Psi(t)⟩] + \sum_{j=1}^{2} \gamma_j (|\hat{A}_j| |\Psi(t)⟩ + |\Psi(t)⟩ |\hat{A}_j⟩).
\]

We are only concerned relaxation from \( |0⟩ \rightarrow |1⟩ \), described by \( \hat{A} = |0⟩⟨1| \) and \( \gamma = \gamma_1 + \gamma_2 \). In the two-qubit systems, this becomes \( \hat{A}_1 = |0⟩⟨1| + i |1⟩⟨0| \).

Optimization in Liouville Space

Unitary target \( \hat{U} \) now implies optimization of \( \rho_0 \rightarrow \hat{U} |\rho_0⟩ |\Upsilon⟩ \) for full set of Liouville basis states \( |\rho_0⟩ \), with the Krotov update formula for \( \Delta \):
\[
\Delta = \frac{\delta S}{\delta \Delta} \bigg|_{\psi_{\text{free}}} = \frac{\delta}{\delta \Delta} \int \left( \sum_{k=1,2} \langle \phi_k(t) | \hat{H}_0 | \phi_k(t) \rangle + \frac{1}{2} \sum_{k=1,2} \langle \phi_k(t) | \hat{H}_1 | \phi_k(t) \rangle + \frac{1}{2} \sum_{k=1,2} \langle \phi_k(t) | \hat{H}_2 | \phi_k(t) \rangle \right) \text{d}t.
\]

Optimization Results in the Dissipative System

Comparison of \( \hat{U} \) directly derived in Liouville space versus optimized \( \hat{U} \) but both approaches are viable. However, convergence for choices of \( \hat{U} \) constructed from Hilbert space states cannot be guaranteed mathematically, especially in the case of non-unitary evolution.

Optimization under dissipation was done with the optimal pulses in the closed space in general pulses. The optimization has limited success in constructing the dissipative. The optimized pulses tend toward bang-bang type, which are limited in their numeral representability, due to the limited finite grid.

We expect that for longer pulse durations, adiabatic dynamics – the other known solution [6, 7] – avoiding dissipation – could be found.

Conclusions & Outlook

We have formulated three approaches to use the Krotov algorithm for gate optimization on a dissipative system. Either the use of fidelity defined directly in Liouville space with an associated \( \hat{U} \), or the construction of \( \hat{U} \) from the states of the closed Hilbert space, in two variants.

For gate optimization, the definition of the functional is only rigorous if the unitary evolution associated with an evolution in Liouville space can be extracted. In general, it remains an open question how to do this.

A continuing investigation of the optimization of the Rydberg gate under dissipation will allow whether optimal control will be able to find non-trivial solutions that go beyond either bang-bang or adiabatic behavior.