Abstract

Optimal control theory provides a powerful method for the realization of entangling two-qubit gates under unitary evolution. This has been demonstrated e.g. for the implementation of a CPHASE gate for trapped neutral atoms [1, 2, 3]. In reality, however, every physical system suffers from decoherence. This could be relaxation due to spontaneous decay or dephasing due to noisy external fields. We therefore consider quantum systems whose dynamics is described by a master equation in Lindblad form. The Krotov optimization method [4] is adapted to operate in Liouville space so that the effect of decoherence is actively taken into account. We formulate the minimum number of optimization targets necessary to implement a unitary gate on the Hilbert space of the system, eliminating all redundancies of the density matrix description. The framework is applied to trapped neutral atoms, using realistic decoherence parameters.

1 Qubit Encoding in the Rydberg System

In RWA: \( \hat{H}_2(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \) with detuning \( \Delta_2 \) for \( |0\rangle \rightarrow |1\rangle, \Delta_3 \) for \( |1\rangle \rightarrow |0\rangle \)

\[ \hat{H}_2 = \hat{H}_2 \otimes I + I \otimes \hat{H}_2 (\alpha r) (\hat{r} r) = \begin{pmatrix} \Delta_2 & \Delta_3 \\ -\Delta_3 & \Delta_2 \end{pmatrix} \]

System Parameters [2]

- \( \Delta_2 = 600 \text{ MHz} \)
- \( \Delta_3 = 1200 \text{ MHz} \)
- \( T = 150 \text{ ns} \)
- \( \gamma = 1 \text{ ns}^{-1} \)

Pulse Parameters

- \( T = 50 \text{ ns}, 150 \text{ ns} \)
- \( \tau = 1, 2 \mu s = 12 \)

2 Hilbert Space Optimization: Krotov Method

Minimum \( F(|\psi(t)\rangle) = -\langle \hat{H}_2(T)|\psi(t)\rangle + \int_0^T \langle \hat{H}_2(t)|\dot{\psi}(t)\rangle dt \)

Update formula [6] for \( \Delta = \psi(-\frac{1}{\gamma}) \)

\[ \Delta = \psi(\frac{1}{\gamma}) = \sum_{k=1}^{N} \langle \phi_k(0)| \hat{O}_k |\psi(0)\rangle \hat{O}_k |\psi(0)\rangle \]

with forward-propagated \( |\phi_k(t)\rangle = \hat{U}(t)|\phi_k(0)\rangle \) and backward-propagated \( |\psi_k(t)\rangle \) with \( |\psi_k(T)\rangle \)

3 Decoherence in the Rydberg System

Master Equation in Lindblad form:

\[ \frac{\partial}{\partial t} |\psi(t)\rangle = -i [\hat{H}_2, |\psi(t)\rangle] + \sum_{A} \left( \hat{A}_A |\psi(t)\rangle \right) \left( \hat{A}_A^\dagger |\psi(t)\rangle \right) \]

Only consider relaxation from \( |i\rangle \rightarrow |0\rangle \)

- Single qubit: \( A = |0\rangle \)
- Two qubits: \( A_1 = |01\rangle, A_0 = |10\rangle \)

4 Optimization in Liouville Space

\[ \Delta(t) = \psi(\frac{1}{\gamma}) = \sum_{k=1}^{N} \langle \phi_k(0)| \hat{O}_k |\psi(0)\rangle \hat{O}_k |\psi(0)\rangle \]

Questions:

1. How many \( \nu_j \) need to be propagated? \( K \) for given Hilbert space dimension \( N \)?
2. How do the \( \xi_j \) have to be constructed?

Optimization Results for a Rydberg Gate

5 Conclusions & Outlook

We have shown that optimal control in Liouville space requires propagation of only two density matrices. This presents a significant speedup compared to previous approaches requiring propagation of \( N^2 \) density matrices.

Using \( F_L \), we have optimized a CPHASE Rydberg gate for varying pulse durations and two different detunings, taking into account decay from \( |i\rangle \rightarrow |0\rangle \). The resulting optimized pulses show bang-bang behavior for short gate times, and adiabatic behavior for long gate times. Using a larger detuning yields a suppression in the decoying state, and thus smaller errors for short pulse durations. The optimization success is comparable with earlier work [2, 3] in which the population in the decoying state was minimized to avoid distillation.

In pending work, we examine the use of the local invariants functional, where the optimization via unitary reconstruction is essential. Besides a dramatic increase in efficiency, we expect an improvement in the gate fidelity due to the additional freedom given by the local equivalence class [2]. The optimization methods presented here are applicable to a wide range of other systems suffering from decoherence.