Two Transmon Qubits Coupled via Cavity Bus

For each point \( (\omega_1, \omega_2) \): find pulse to minimize entanglement (two-qubit gate) and pulse to implement local gate \( \in \text{SU}(2) \otimes \text{SU}(2) \), using multistage optimization scheme [5].

### Parameters
- \( \omega_1 = 6.0 \text{ GHz} \)
- \( \omega_2 = 5.8 \div 7.5 \text{ GHz (var)} \)
- \( \delta_1 = 4.5 \div 11.0 \text{ GHz (var)} \)
- \( \gamma_1 = 290 \text{ MHz} \)
- \( \gamma_2 = 310 \text{ MHz} \)
- \( q = 70 \text{ MHz} \)

\[
\hat{\mathbf{H}} = \frac{\hbar}{2} \sum_{j=1}^{2} \left[ \omega_j \hat{b}_j + \frac{g_j}{2} \hat{a} \cdot \hat{b}_j + \frac{g_j}{2} \hat{b} \cdot \hat{a} \right] + \frac{\alpha_j}{2} \hat{b}^\dagger \hat{b} + \frac{\alpha_j}{2} \hat{a}^\dagger \hat{a}
\]

(1)

with \( \hat{b} \) and \( \hat{a} \): cavity harmonic oscillators, \( g_j \): qubit anharmonic oscillators, \( g \): qubit-cavity coupling, and \( \alpha_j \): cavity coupling to control field.

\[
e(\tau) = E_0 R(t) \cos(\omega_1 \tau), \quad R(t) = \text{Blackman shape}
\]

(2)

Include spontaneous decay: lifetime of cavity \( \tau_c = 3.2 \mu s \) [4]; lifetime of qubit \( \tau_q = 13.3 \mu s \) [2].

### Method

**Goal:** For each point \( (\omega_1, \omega_2) \): random frequencies \( \omega_2 \) scan amplitude \( E_0 \in [10 \div 900] \text{ MHz} \).

1. **Random Search**
   - For each point \( (\omega_1, \omega_2) \): random frequencies \( \omega_2 \) scan amplitude \( E_0 \in [10 \div 900] \text{ MHz} \).

2. **Gradient-Free Optimization of Analytical Pulse Parameters**
   - For best values of step 1, use Nelder-Mead downhill simplex to minimize Eq. (3) for free pulse parameters \( E_0, \omega_2 \).

3. **Gradient-Based Optimization (Krotov’s method) for Fine-Tuning**
   - Use Krotov’s method [7] to continue optimization of (1) for arbitrary perfect entanglers [8] and arbitrary local gate \( \in \text{SU}(2) \otimes \text{SU}(2) \), based on Cartesian decomposition [10].

### Optimization Success (best obtained values)

- Expected error due to dissipation:
  - \( \epsilon_{\text{PE}} = 1 - \epsilon_{\text{PE}} \frac{\Omega_{\text{Q}}}{\Omega} \)
  - with \( \Omega_{\text{Q}} = \Omega \frac{Q_{\text{Q}}}{Q} = \frac{Q_{\text{Q}}}{Q} \cdot \Omega \).

### Conclusions & Outlook

- Found parameters allowing implementation perfect entangler and local gate, for gate durations down to 10 ns, beating decoherence with gate error \( < 1 \times 10^{-3} \).
- Obtained gates are limited only by dissipation.
- Fastest gates can be achieved in previously under-explored (non-dissipative) parameter regime with \( \epsilon_{\text{PE}} \approx \omega_c \).
- Using gate durations allow wide range of two-qubit gates, for short gate durations, the dissipation is most efficient.
- More complicated pulse shapes than Eq. (2) have been tried, but provide no significant improvement. Outlook: implement complete set of universal quantum gates by directly optimizing single-qubit Hadamard and phase gates.
- Analyze characteristics of optimal pulses and dynamics. What gate mechanisms are used?