Constant Accelerated Rotation

Michael Goerz, Anton Hause

Introduction

Rotational movement: the rotational movement of rigid bodies can be described analoguesly to linear displacement. In this analogy the displacement x corresponds to the angle ϕ, the speed $v = \dot{x}$ corresponds to the angular speed $\omega = \dot{\phi}$, the acceleration $a = \ddot{x}$ corresponds to the angular acceleration $\ddot{\phi}$, the force F corresponds to the torque M, the ~~for~~ mass m corresponds to the moment of inertia I, and the momentum p corresponds to the angular momentum L

That means the equation of movement is

$$M = I \cdot \ddot{\phi} \quad ; \quad M = \dot{L}$$

Torque and angular momentum can also be described by their corresponding quantities of displacements acting over a lever arm.

$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{M} = \vec{r} \times \vec{F}$$

In any closed system, torque is preserved

$$L = \text{const} \Leftrightarrow M = 0$$

The angular momentum accounts for how the mass is distributed in respect to the axis of rotation:

$$I = \int_V r^2 \, dm$$

For a cylinder, rotating around its axis of symmetry, the angular momentum is

$$I = \int_V r^2 \, dm = \int_V r^2 \cdot \rho \cdot dV$$

$$= \int r^3 \cdot \rho \, dr \, d\varphi \, dz = \frac{1}{4} a^4 \rho \cdot h \cdot 2\pi = \frac{1}{2} m a^2,$$

a being the cylinder's radius

If the body is not rotating around its axis of symmetry, Steiner's law can be applied to simplify the calculation

$I = I_s + m a^2$, I_s being the angular momentum in respect to the axis of symmetry and a being the distance to that axis

This can be explained ~~at~~ as follows:

The rotation is split up in two parts. One part is the rotation of the entire body (as a point mass) around the axis of rotation, that's the ma^2 part, the other is the rotation of the body around its own axis of symmetry.

With all this, the solution to the equation of movement can be given as

$$\varphi(t) = \frac{M}{2I} t^2 + \dot{\varphi}_0 t + \varphi_0$$

Assignments

Examination of constant accelerated rotation
- measurement of distance - time - dependancy
- measurement of torque - time - dependancy
- measurement of friction

for different angular momenta (with and without additional masses

1) Qualitative and Quantitave examination of the law of movements. Measurement of time in dependancy of angle (constant torque) and in dependancy of torque (constant angle). Determination of angular momenta (with and without additional masses) and comparison with the theoretic value (Steiner's law)
2) Discussion of friction in different models (dependency of friction forces and friction torques of the parameters of movement) from the experimental data

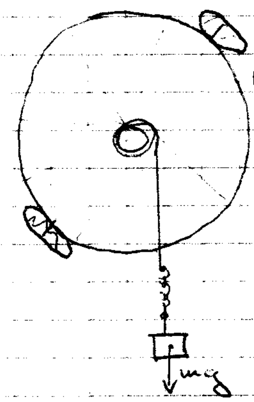
Experiment

Michael Goerz, Anton Haase

Tutor: Lisong Yang

9.3.05 Start 9^{45} End

Devices:



Handclock (1/10s)

set of weights

mass of spring (10 ± 1 g)

geometry: Diameter wheel 63,5 ± 0,5 cm

Diameter axis 3,5 ± 0,2 cm

Diameter ext. weights 5,5 ± 0,5 cm

On Assignment 1:

M = const m = (89 ± 1) g

~~mass~~ mass of weight and spring (99 ± 1) g

left-rotation

$\phi/2\pi$	t
1	(7,7 ± 0,5) s
2	(11,5 ± 0,5) s
3	13,1
4	15,3
5	17,1
6	/

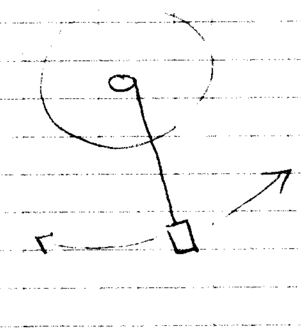
error of t?

- reaction time
- precision of ϕ

not enough data

error:

(in general)



$m = 89\,g$

right-rotation ↻

$\phi/2\pi$	t
1	6,7 s
2	9,8
3	~~11,4~~ 12,9
4	14,7
5	16,0
6	17,9
7	19,5

$m = 89\,g$ ↺

$\phi/2\pi$	
1	7,5
2	10,3
3	12,9
4	15,0
5	17,0
6	18,6
7	20,2

with additional wheights

mass of weight and spring : 249 ± 1 g

additional wheigts 1010 ± 1 g each

$\phi / 2\pi$	t
1	8,5
2	12,5
3	15,43
4	18,0
5	20,1
6	23,0
7	24,2

$\phi / 2\pi$	t
1	10,4
2	13,6
3	17,5
4	18,9
5	21,8
6	23,4
7	25,7

$\phi = \text{const} = 3$

m	t	
100 g	13,2	
249 g	7,9	
527 g	5,4	
735 g	4,5	4,9
997 g	4,0	

m	t
100 g	12,5
249 g	7,8
527 g	5,4
735 g	4,5
997 g	4,0

wheights include spring

all wheights ± 1 g

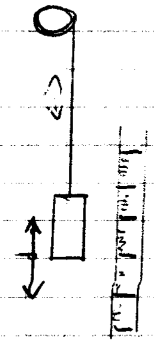
with additional weights 1010g each

$\phi = 5$

m	t
100 g	32,3
249 g	21,7
527 g	14,3
735 g	12,2
997 g	10,3

m	t
100 g	(39,5)
249 g	21,2
527 g	14,8
735 g	12,2
997 g	10,5

On Assignment 2:



x_0 = initial position

x_1 = position after down-up

x_0	x_1	m	ext. mass
70	51,5	249 g	1010 g each ~~all values 30 cm~~
30	23,5	249g	"
70	57,5	527g	"
30	24,5	527g	"
70	51,5	249g	0
30	24,5	249g	"
70	51,0	527g	"
30	24,5	527g	"

[illegible] 09/03/05

on Assignment 2:

~~Zero position : 2433 ± 1~~ experiment repeated

~~Start position : 100 ± 1~~

x_0	x_1	m	ϕ ±0,2r	ext. mass
100	86	249	10 −0,9rad	1010
99	85	249	9 + $\frac{3}{4}$	1010
93	84	527	10 +0,6rad	1010
93	84	527	10+0,6rad	1010
74	65	249	6 +0,6rad	1010
74	65	249	6+0,7rad	1010
69,5	63	527	6$\frac{1}{2}$ − 0,4r	1010
69	64	527	6$\frac{1}{2}$ −0,2r	1010
99	76	249	9$\frac{1}{4}$	0
99 ~~98,5~~	78	249	9$\frac{1}{4}$ −0,2r	0
95	81	527	10 −0,8r	0
93	80	527	9$\frac{3}{4}$ +0,1r	0
74,5	63	249	6 − 0,2r	0
74	61	249	5$\frac{3}{4}$ +0,1r	0
73	65,5	527	6$\frac{1}{2}$ −0,6r	0
72	64,5	527	6$\frac{1}{4}$ +0,2r	0

Yang
11/03/05

Analysis

Assignment 1:

The first measurement, with a fixed mass and variable angles, allows to determine the moment of inertia. Neglecting friction, we know that the total moment of inertia is

$I_1 = I_0 + I_w$, I_0 being the moment of inertia of the wheel without additional weights and I_w being the moment of inertia of the additional weights.

Since we do not know all details about the wheel, we will only make a comparison of the experimental and the theoretical value of I_w

In theory, we should find that

$$I_w = 2m\left(\frac{1}{2}r^2 + a^2\right) \quad \text{(following Steiner's law)}$$

$$= 2\,(1{,}010 \pm 0{,}001)\,kg \cdot \left(\frac{1}{2}(0{,}0275 \pm 0{,}0025)^2 m^2 + (0{,}3175 \pm 0{,}0025)^2 m^2\right)$$

$$= (0{,}204 \pm 0{,}015)\,kg^2\,m^2$$

For the experimental value, we can use

$$M = 2I\phi \cdot \frac{1}{t^2}$$

$$\Leftrightarrow I = \frac{F_g \cdot d}{2} \cdot \frac{t^2}{\phi}$$

The plot gives the inverse ratio, $\frac{\phi}{t^2}$

In the first plot, with fixed mass 89g,

$$\frac{\phi}{t^2} = (0{,}1100 \pm 0{,}0038)\,s^{-2}$$

$$\Rightarrow I_0 = \frac{1}{2}(0{,}0153 \pm 0{,}0009)\,Nm \cdot \frac{1}{(0{,}1100 \pm 0{,}0038)\,s^{-2}}$$

$$= \frac{1}{2}(0{,}139 \pm 0{,}010)\,Nms^{2} = (0{,}070 \pm 0{,}005)\,Nms^2$$

In the second plot, with fixed mass 249 g,

$$\frac{\phi}{t^2} = (0{,}0784 \pm 0{,}0154)\ s^{-2}$$

$$\Rightarrow I_1 = \frac{1}{2}(0{,}0427 \pm 0{,}0003)\ Nm \cdot \frac{1}{(0{,}0784 \pm 0{,}0154)\ s^{-2}}$$

$$= \frac{1}{2}(0{,}545 \pm 0{,}108)\ Nms^2 = (0{,}273 \pm 0{,}054)\ Nms^2$$

We now get I_w as

$$I_w = I_1 - I_0 = \frac{1}{2}(0{,}406 \pm 0{,}109)\ Nms^2 = (0{,}203 \pm 0{,}055)\ Nms^2$$

The second measurement, with a fixed angle and variable masses (i.e. torques) also allows us to look at the friction. We can use the law

$$M = M_r + 2\phi \cdot \frac{I}{t^2}$$

$$\Leftrightarrow m \cancel{gd} = m_r \cancel{gd} + 2\phi \cdot \frac{I_0 + I_w}{t^2} \cdot \frac{1}{gd}$$

$$\Leftrightarrow m \cdot t^2 = m_r t^2 + \underbrace{\frac{2\phi(I_0 + I_2)}{g \cdot d}}_{\text{const} := C}$$

$$\Leftrightarrow (m - m_r) t^2 = C$$

$$\Leftrightarrow \frac{m - m_r}{1/t^2} = C \qquad \Leftrightarrow \qquad m = C \cdot \frac{1}{t^2} + m_r$$

Thus, the plot's slope gives us C

In the first plot, with fixed angle 6π, the slope as calculated by gnuplot is

$$C = (15{,}852 \pm 5{,}873)\ kgs^2$$

$$\Rightarrow I_0 = \frac{g \cdot d \cdot C}{2\phi} = \frac{1}{2}(0{,}144 \pm 0{,}054)\ Nms^2$$

$$= (0{,}072 \pm 0{,}027)\ Nms^2$$

In the second plot, with fixed angle 10π, we have

$$c = (106{,}09 \pm 11{,}90)\ kg\,s^2$$

$$\Rightarrow I_1 = \frac{g \cdot d \cdot c}{2\phi} = \frac{1}{2}(0{,}580 \pm 0{,}073)\ Nm\,s^2$$
$$= (0{,}290 \pm 0{,}037)\ Nm\,s^2$$

From these two values, we get

$$I_\omega = I_1 - I_0 = \frac{1}{2}(0{,}436 \pm 0{,}091)\ Nm\,s^2 = (0{,}218 \pm 0{,}046)\ Nm\,s^2$$

M_r from the plot indicates the friction

$$M_r = m_r \cdot g \cdot d$$

In the first plot, m_r is calculated as

$$m_r = (-0{,}00574 \pm 0{,}01556)\ kg$$

$$\Rightarrow M_r = (-9{,}85 \pm 26{,}70) \cdot 10^{-4}\ kg\frac{m^2}{s^2}$$

M_r must be greater than zero, so M_r can be anywhere between 0 and $16{,}89 \cdot 10^{-4}\ kg\frac{m^2}{s^2}$

In the second plot, m_r is

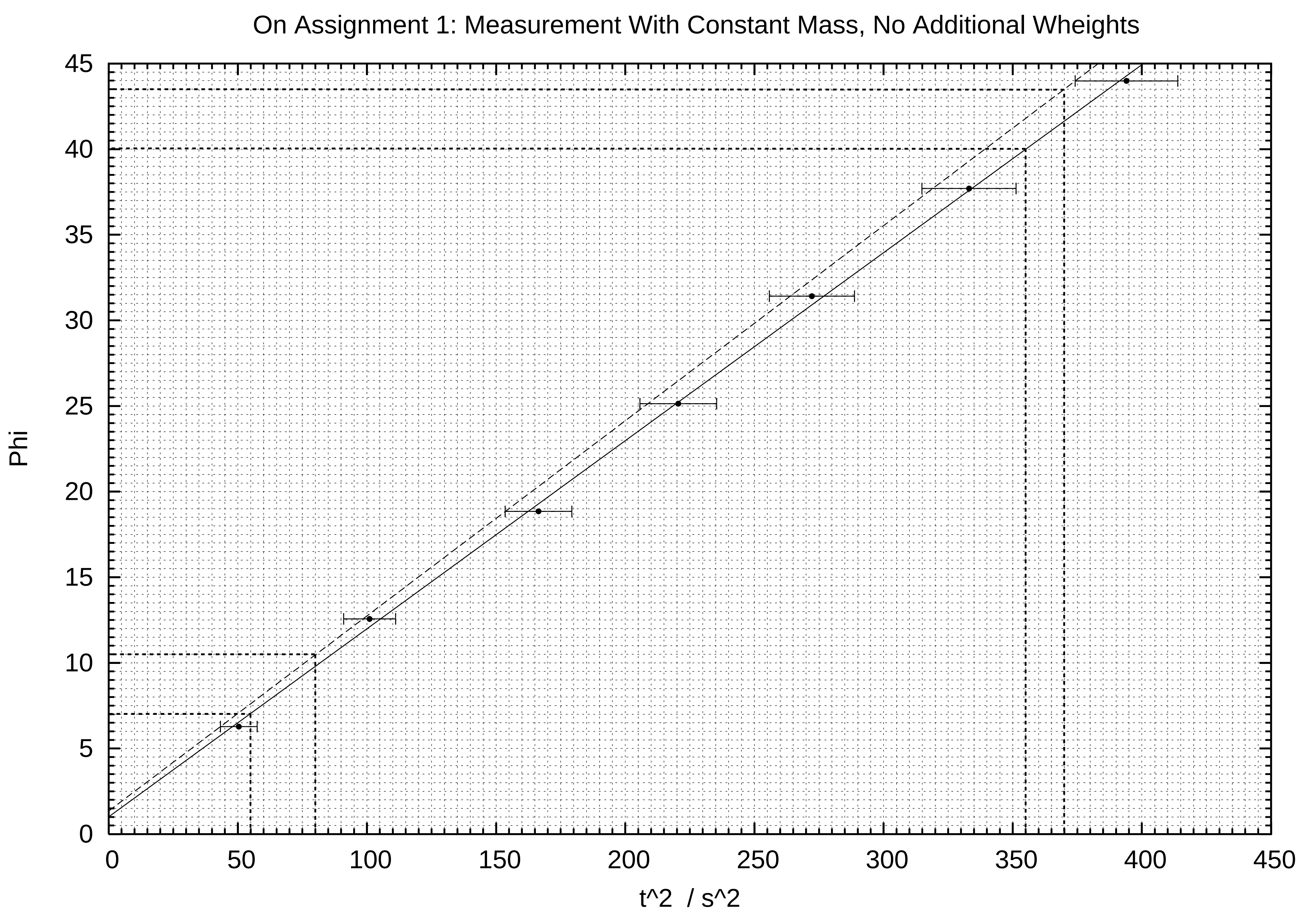
$$m_r = (0{,}02005 \pm 0{,}00705)\ kg$$

$$\Rightarrow M_r = (29{,}50 \pm 13{,}32) \cdot 10^{-4}\ kg\frac{m^2}{s^2}$$

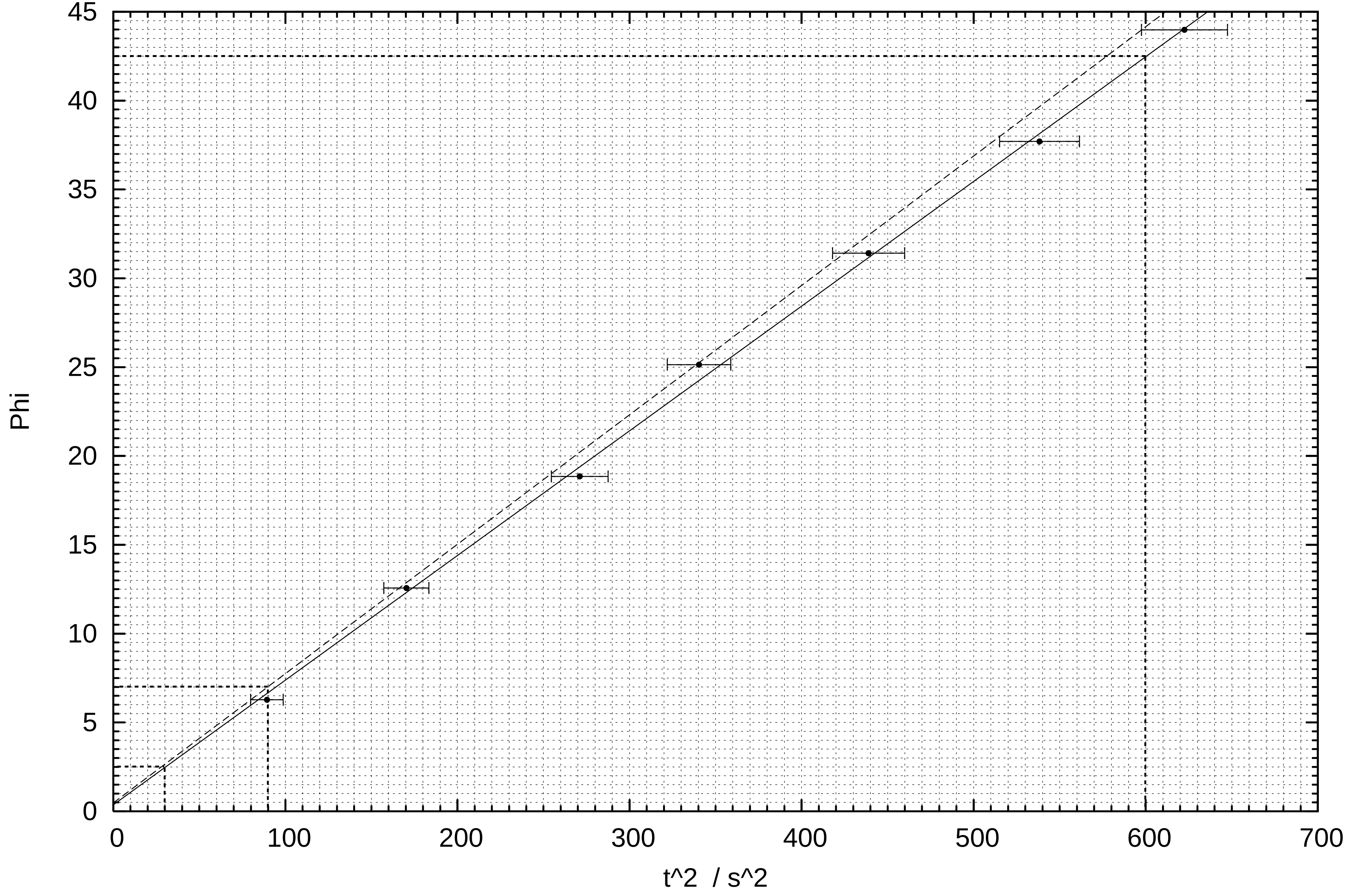
Considering the large error, these values have little meaning.

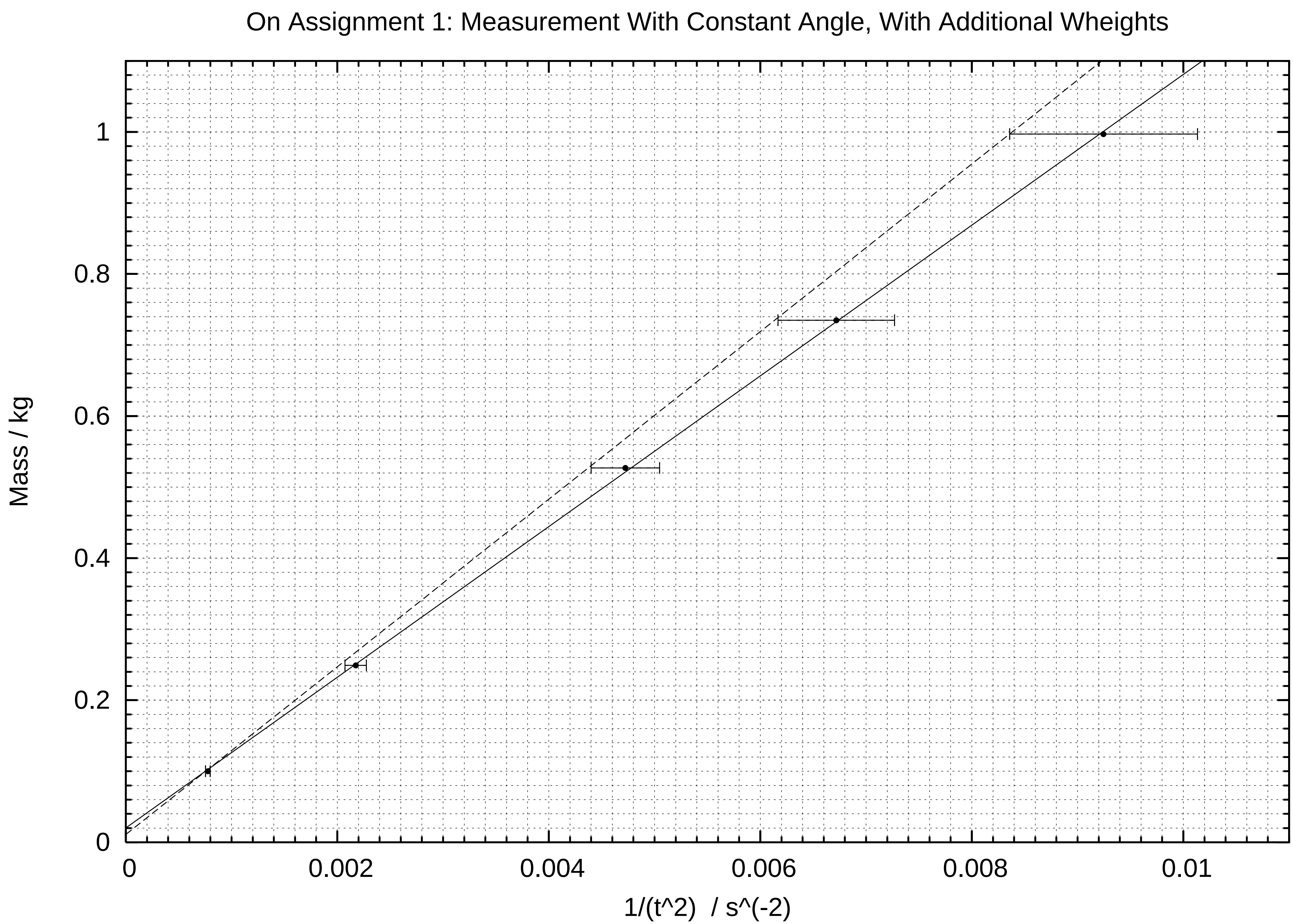
On Assignment 1: Experiment and Plot Data

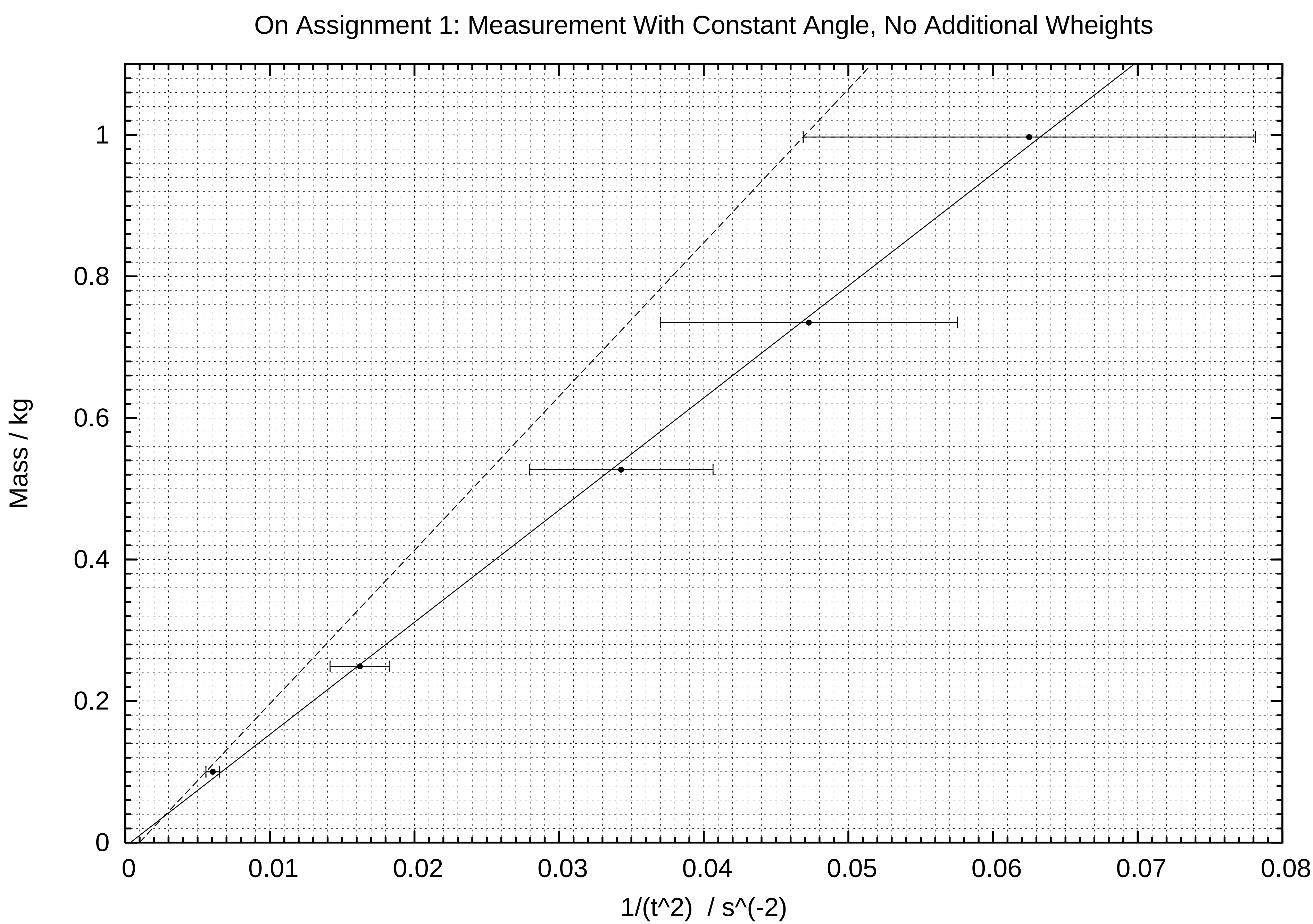
Rounds	Phi (rad)	Time (left spinning)	Time (right spinning)	Average Time	Error	Mass	Error	Ext. Mass	Error	Time^2	Error	Inverse	Error
1	6.28	6.7	7.5	7.1	0.5	0.099	0.001			50.41	7.10	0.019837	0.002794
2	12.57	9.8	10.3	10.1	0.5	0.099	0.001			101.00	10.05	0.009901	0.000985
3	18.85	12.9	12.9	12.9	0.5	0.099	0.001			166.41	12.90	0.006009	0.000466
4	25.13	14.7	15.0	14.9	0.5	0.099	0.001			220.52	14.85	0.004535	0.000305
5	31.42	16.0	17.0	16.5	0.5	0.099	0.001			272.25	16.50	0.003673	0.000223
6	37.7	17.9	18.6	18.3	0.5	0.099	0.001			333.06	18.25	0.003002	0.000165
7	43.98	19.5	20.2	19.9	0.5	0.099	0.001			394.02	19.85	0.002538	0.000128
1	6.28	8.5	10.4	9.5	0.5	0.249	0.001	1.010	0.001	89.30	9.45	0.011198	0.001185
2	12.57	12.5	13.6	13.1	0.5	0.249	0.001	1.010	0.001	170.30	13.05	0.005872	0.000450
3	18.85	15.4	17.5	16.5	0.5	0.249	0.001	1.010	0.001	271.10	16.47	0.003689	0.000224
4	25.13	18.0	18.9	18.5	0.5	0.249	0.001	1.010	0.001	340.40	18.45	0.002938	0.000159
5	31.42	20.1	21.8	21.0	0.5	0.249	0.001	1.010	0.001	438.90	20.95	0.002278	0.000109
6	37.7	23.0	23.4	23.2	0.5	0.249	0.001	1.010	0.001	538.24	23.20	0.001858	0.000080
7	43.98	24.2	25.7	25.0	0.5	0.249	0.001	1.010	0.001	622.50	24.95	0.001606	0.000064
3	18.85	13.2	12.5	12.9	0.5	0.100	0.001			165.12	12.85	0.006056	0.000471
3	18.85	7.9	7.8	7.9	0.5	0.249	0.001			61.62	7.85	0.016228	0.002067
3	18.85	5.4	5.4	5.4	0.5	0.527	0.001			29.16	5.40	0.034294	0.006351
3	18.85	4.7	4.5	4.6	0.5	0.735	0.001			21.16	4.60	0.047259	0.010274
3	18.85	4.0	4.0	4.0	0.5	0.997	0.001			16.00	4.00	0.062500	0.015625
5	31.42	32.3	39.5	35.9	0.5	0.100	0.001	1.010	0.001	1288.81	35.90	0.000776	0.000022
5	31.42	21.7	21.2	21.5	0.5	0.249	0.001	1.010	0.001	460.10	21.45	0.002173	0.000101
5	31.42	14.3	14.8	14.6	0.5	0.527	0.001	1.010	0.001	211.70	14.55	0.004724	0.000325
5	31.42	12.2	12.2	12.2	0.5	0.735	0.001	1.010	0.001	148.84	12.20	0.006719	0.000551
5	31.42	10.3	10.5	10.4	0.5	0.997	0.001	1.010	0.001	108.16	10.40	0.009246	0.000889


On Assignment 1: Measurement With Constant Mass, No Additional Wheights
Phi
0
5
10
15
20
25
30
35
40
45
0
50
100
150
200
250
300
350
400
450
t^2 / s^2

On Assignment 1: Measurement With Constant Mass, With Additional Wheights




On Assignment 1: Measurement With Constant Angle, With Additional Wheights
Mass / kg
0
0.2
0.4
0.6
0.8
1
0
0.002
0.004
0.006
0.008
0.01
1/(t^2) / s^(-2)



On Assignment 1: Measurement With Constant Angle, No Additional Wheights
Mass / kg
1
0.8
0.6
0.4
0.2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
1/(t^2) / s^(-2)

Assignment 2

In this assignment, friction is described as a constant friction torque. It can be calculated from the loss of energy. If the mass starts at a position x_0 and returns to a position x_1, then the difference of energy is $E = m \cdot g \cdot \Delta x$. We consider this energy to have gone into a friction torque, for which we have the equation

$$W_r = E = M_r \cdot \phi$$

Thus, we can find the friction torque from the experimental data as

~~$M_r = m \cdot g \cdot \Delta x$~~

$$M_r = m \cdot g \cdot \Delta x / \phi$$

The calculated values are in the table.
We can see that M_r is neither constant nor a constant percentage of the torque M. However, M_r is much larger without the external wheights. This indicates that M_r is dependant on the total angular momentum
Obviously, we must include more parameters into a description of friction

On Assignment 2: Measurement of Friction

X_0	Error	X_1	Error	Delta x (cm)	Error	Mass (kg)	Error	Ext. Mass (kg)	Error	Full Rots (rad / 2Pi)	+ rad	Total Phi (rad)	Error	M (N m)	M_r (N m)	Error	N_r / M
100	1	86	1	14	1.41	0.249	0.001	1.010	0.001	10	-0.9	61.93	0.2	0.043	0.006	0.001	0.13
99	1	85	1	14	1.41	0.249	0.001	1.010	0.001	9.75	0	61.26	0.2	0.043	0.006	0.101	0.13
93	1	84	1	9	1.41	0.527	0.001	1.010	0.001	10	0.6	63.43	0.2	0.090	0.007	0.157	0.08
93	1	84	1	9	1.41	0.527	0.001	1.010	0.001	10	0.6	63.43	0.2	0.090	0.007	0.157	0.08
74	1	65	1	9	1.41	0.249	0.001	1.010	0.001	6	0.6	38.3	0.2	0.043	0.006	0.157	0.13
74	1	65	1	9	1.41	0.249	0.001	1.010	0.001	6	0.7	38.4	0.2	0.043	0.006	0.157	0.13
69.5	1	63	1	6.5	1.41	0.527	0.001	1.010	0.001	6.5	-0.4	40.44	0.2	0.090	0.008	0.218	0.09
69	1	64	1	5	1.41	0.527	0.001	1.010	0.001	6.5	-0.2	40.64	0.2	0.090	0.006	0.283	0.07
99	1	76	1	23	1.41	0.249	0.001	0.000	0.001	9.25	0	58.12	0.2	0.043	0.010	0.062	0.23
99	1	78	1	21	1.41	0.249	0.001	0.000	0.001	9.25	-0.2	57.92	0.2	0.043	0.009	0.068	0.21
95	1	81	1	14	1.41	0.527	0.001	0.000	0.001	10	-0.8	62.03	0.2	0.090	0.012	0.101	0.13
93	1	80	1	13	1.41	0.527	0.001	0.000	0.001	9.75	0.1	61.36	0.2	0.090	0.011	0.109	0.12
74.5	1	63	1	11.5	1.41	0.249	0.001	0.000	0.001	6	-0.2	37.5	0.2	0.043	0.007	0.123	0.18
74	1	61	1	13	1.41	0.249	0.001	0.000	0.001	5	0.1	31.52	0.2	0.043	0.010	0.109	0.24
73	1	65.5	1	7.5	1.41	0.527	0.001	0.000	0.001	6.5	-0.6	40.24	0.2	0.090	0.010	0.189	0.11
72	1	64.5	1	7.5	1.41	0.527	0.001	0.000	0.001	6.35	0.2	40.1	0.2	0.090	0.010	0.189	0.11

Conclusion

In general, the wheel followed the law of movement as expected. The exprimental values of I_ω are within error identical to the theoretical value. However, the errors are extremely large. With the given setup, it will be hard to get much more accurate results, as there are numerous sources of error: There a swinging movement of the mass during certain measurements, there is a slightly uneven mass distribution on the wheel, and there is friction both from the wheel and from the air.

The attempt to analyze the friction in assignment 1 were rather unsuccessful, they merely provided an upper bound from the error interval.

In Assignment 2, we could make some meaningful measurement of friction. The results show that we cannot describe friction ~~with~~ accurately with simple models, but that we need more parameters. A source of error in this assignment was, in addition to the sources mentioned above, the loss of energy at the point of full extension.