Ball-Drop-Viscosimeter

Michael Goerz, Anton Haase

Introduction

The behaviour of fluids is strongly influenced by the amount of inner friction. If the inner friction is high, there is a laminar flow, i.e. thin layers of fluid flow smoothly over each other. If the inner friction is low, however, there is a turbulent flow in which thin layers of fluid curl and mix. The amount inner friction is described by the viscosity η. It is defined as a coefficient in the relation between the force one has to apply to "pull" on the surface and the velocity of that top surface relative to the bottom.
If dz is the thickness of that surface, and A is the area, then

$$F = \eta \cdot A \frac{dv}{dz}$$

Connected to the viscosity is the Reynolds number, which decides whether the flow is laminar or turbulent.

$$Re := \frac{\varrho v l}{\eta}$$, v being the flow's velocity and l being the length of the flow

For small Reynolds numbers, the flow is laminar,
Typically, in pipes, the shift between

laminarity and turbulance occurrs between 2000 and 4000.

When a ball is dropped in a fluid with high viscosity, there will be laminar friction against its fall. According to the rule of Stokes

$F_R = -6\pi\eta v r$

Because ~~I~~ the friction is proportional to the speed, the friction will reach an equilibrium with the gravitational force as the ball is accelerated and the ball will then ~~follow~~ continue to fall at a constant speed. In the equation of movement, we also have to take into account the lift.

$$m \cdot \dot{v} = \frac{4}{3}\pi r^3 (\rho_K - \rho_{oil}) g - 6\pi\eta r \cdot v \quad \Big| \cdot \frac{1}{dv}$$

$$\Leftrightarrow m \cdot \frac{1}{dt} = \left[\frac{4}{3}\pi r^3 g \Delta\rho - \frac{18}{3}\pi\eta r \cdot v\right] \cdot \frac{1}{dv}$$

$$\Leftrightarrow \int \frac{1}{m} dt = 3 \int \frac{1}{4\pi r^3 g \Delta\rho - 18\pi\eta r v} dv$$

$$\Leftrightarrow \frac{t}{m} = \cancel{3}^{1} \cdot \frac{-1}{\underset{6}{\cancel{18}}\pi r \eta} \cdot \ln\left(-18\pi r v \eta + 4\pi r^3 g \Delta\rho\right) + C$$

$$\Leftrightarrow e^{\frac{-6\pi r\eta}{m} t} = (4\pi r^3 g \Delta\rho - 18\pi r v \eta) \cdot C$$

$$\Leftrightarrow v = \frac{-1}{18\pi r \eta \cdot C}\left[e^{\frac{-6\pi r\eta}{m} t} - 4 \cdot C \cdot \pi r^3 g \Delta\rho\right]$$

with $m = \frac{4}{3}\pi r^3 \cdot \rho_K$

$$\Leftrightarrow v = \frac{-1}{18\pi r\eta \cdot C}\left[e^{\frac{-9\eta}{2\rho_K r^2} t} - 4 \cdot C \cdot \pi r^3 g \Delta\rho\right]$$

The condition is that $v(t=0) \overset{!}{=} 0$, which gives C as

$$C = \frac{1}{4\pi r^3 g (\rho_K - \rho_{oil})}$$

The law for the velocity is then

$$v(t) = \frac{-2g \cdot r^2 (\rho_K - \rho_{air})}{9\eta} \cdot \left(e^{\frac{-9\eta}{2\rho_K r^2} \cdot t} - 1 \right)$$

The exponential term will quickly decrease towards zero, if the viscosity is high enough.
The resulting constant speed is

$$v_\infty = \frac{2 g r^2 \Delta\rho}{9\eta}$$

For the initial phase of acceleration, we can calculate the time until the velocity reaches 99,9% of the final velocity as

$$t_\infty = \frac{-2\rho_K \cdot r^2}{9\eta} \cdot \ln(0{,}001)$$

Additionally, we know that the viscosity is dependant on temperature as

$$\eta(T) = A \cdot e^{\frac{B}{T}}$$

If we enter that into the equation for the final velocity, we can find a formula for time we can exspect the ball to need to dropp a given distance h, in dependancy of temperature:

$$t\left(\tfrac{1}{T}\right) = \underbrace{\frac{9 \cdot A \cdot h}{2 g r^2 \Delta\rho}}_{t_0} \cdot e^{B \frac{1}{T}}$$

Assignments

1) Measurement of the drop velocity of steel balls with known and unknown radius in dependancy of temperature

2) Examination of the relation between the ricine oil's viscosity and the temperature
Determination of the oil's viscosity at 20° C and comparison with the literature value

3) Determination of the unkown balls' radii from the experiment data and comparison with the value obtained from direct measurement

4) Formulation of the law of movement and solution with the initial condition $v(t=0) = 0$.
Approximate calculation of the time or the drop distance until the balls reach their final velocity.

Experiment

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29.3.05 start 10^{00} end 13^{00}

Devices:

cylinder with ricine oil

steel ball with known radius $r_0 = 1{,}000 \pm 0{,}002$

steel balls with unknown radii r_a, r_b, r_c

density of the ricine oil:

$$\rho_{oil} = (0{,}975 \pm 0{,}005) \cdot 10^3 \ kg/m^3$$

density of steel:

$$\rho_K = (7{,}81 \pm 0{,}02) \cdot 10^3 \ kg/m^3$$

$$\Rightarrow \Delta\rho = \rho_K - \rho_{oil} = (6{,}84 \pm 0{,}03) \cdot 10^3 \ kg/m^3$$

stopwatch

micrometer screw

drop distance $h = (18{,}0 \pm 0{,}1) \cdot 10^{-2} m$

laboratory temperature : $24{,}2°C$

conversin celsius → kelvin : $0°C = 273{,}2 \ K$

Measured ball radii:

$$2r_a = (1{,}45 \pm 0{,}002) \ mm$$

$$2r_b = (1{,}52 \pm 0{,}002) \ mm$$

$$2r_c = (2{,}47 \pm 0{,}002) \ mm$$

$$2r_0 = (1{,}97 \pm 0{,}002) \ mm$$

measurements:

T_1 / °C	T_2 / °C	ball radius	drop time / sec
4,7	5,9	r_0	67,7
6,5	(8,2)	r_a	225,0
7,4	8,6	r_b	90,6
8,6	9,3	r_c	37,0
9,6	9,9	r_0	54,6
10,3	10,9	r_a	172,4
11,1	11,3	r_b	70,9
11,4	11,5	r_c	28,6
11,7	11,8	r_0	42,6
11,9	12,2	r_a	~~170,4~~ 140,4
12,3	12,4	r_b	57,3
12,5	12,6	r_c	23,7
12,6	12,7	r_0	35,0
12,9	13,1	r_a	123,3
13,2	13,4	r_b	49,2
13,5	13,6	r_c	20,2
13,7	13,7	r_0	30,3
13,8	14,0	r_a	108,3
14,2	14,4	r_b	43,0
14,5	14,5	r_c	17,9
14,6	14,6	r_0	26,9
14,7	15,0	r_a	96,9
15,1	15,2	r_b	44,4
15,3	15,3	r_c	16,2
15,4	15,4	r_0	24,5

T_1 / °C	T_2 / °C	ball radius	drop time / sec
15,6	15,7	r_a	89,2
15,8	15,9	r_b	36,2
16,0	16,0	r_c	15,3
16,2	16,2	r_o	22,3
16,3	16,5	r_a	85,4
16,5	16,6	r_b	37,7
16,7	16,7	r_c	13,8
16,8	16,8	r_o	21,0
17,0	17,1	r_a	85,0
17,1	17,2	r_b	31,6
17,3	17,3	r_c	13,1
17,4	17,4	r_o	19,9
17,5	17,6	r_a	76,1
17,7	17,7	r_b	33,8
17,8	17,8	r_c	12,5
17,9	17,9	r_o	18,7
18,0	18,1	r_a	71,8
18,1	18,2	r_b	28,8
18,2	18,2	r_c	12,0
18,3	18,3	r_o	18,3
18,4	18,5	r_a	67,9
18,5	18,6	r_b	27,6
18,6	18,6	r_c	11,6
18,9	18,9	r_o	17,2
18,7	18,8	r_a	66,5

T_1 /°C	T_2 /°C	ball radius	drop time /sec
18,9	19	r_b	30,0
19,1	19,1	r_c	11,0
19,1	19,1	r_0	17,0
19,1	19,2	r_a	64,2
19,4	19,5	r_b	25,5
19,6	19,6	r_c	10,5
19,7	19,7	r_0	16,0
19,7	19,8	r_a	60,3
19,9	19,9	r_b	24,5
20,0	20,0	r_c	10,2
~~23,0~~ 20,3	~~23,0~~ 20,3	r_0	15,2
~~23~~, 20,4	20,5	r_a	59,2
20,5	20,6	r_b	23,0
20,6	20,6	r_c	9,9
20,7	20,7	r_0	14,8
20,8	20,9	r_a	56,0
20,9	20,9	r_b	~~52,5~~ 22,5
20,9	20,9	r_c	9,4
21,0	21,0	r_0	14,4
21,0	21,1	r_a	55,5
21,1	21,1	r_b	22,0
21,2	21,2	r_c	9,3

Analysis

Assignment 1:

The table shows the individual calculated values for the final velocity. The dependancy on temperature can be seen in the following six plots. For higher accuracy and for practical reasons, we examine the relation of drop time for the given distance of 18 cm and the temperature. The last plot shows the actual values of the velocity, as calculated, for comparison. Of, course, the relation is exactly antiproportional to the previous plots.

From the theory we were expecting an exponential dependancy:

$$t(T) = \frac{9\eta}{2 g r^2 \Delta\rho} s = \underbrace{\frac{9 \cdot A \cdot s}{2 g r^2 \Delta\rho}}_{t_0} \cdot e^{B/T}$$

The plots show very clearly that this formula is generally confirmed by the data

For low temperatures, there is a slightly super-exponential tendency, so we left out some points in that area for the fit

From the slope, we can find a value for B, neglecting the ~~error~~ of the individual data points the values are

$B_0 = 9051$ K

$B_a = 9252$ K *

$B_b = 8592$ K

$B_c = 8717$ K

The value for r_a should be neglected, because some of the balls labeled as r_a actually had diffrent radii. The average of the remaining three values gives

$B = (8786 \pm 265)$ K The error was taken from the difference of the average value to the furthest value. The error is probably much higher in reality, nonetheless our value of B should show the correct order of magnitude

The value of t_0 cannot be obtained from the data, but the calculations for the fit show that it is very small ($< 10^{-10}$)

Assignment 2:

We can find the viscosity directly from the final velocity:

$$v_\infty(t) = \frac{+2gr^2(\varrho_K - \varrho_{oil})}{9\eta}$$

$$\Leftrightarrow \eta(T) = \frac{2gr^2\Delta\varrho}{9s} \cdot t(T)$$

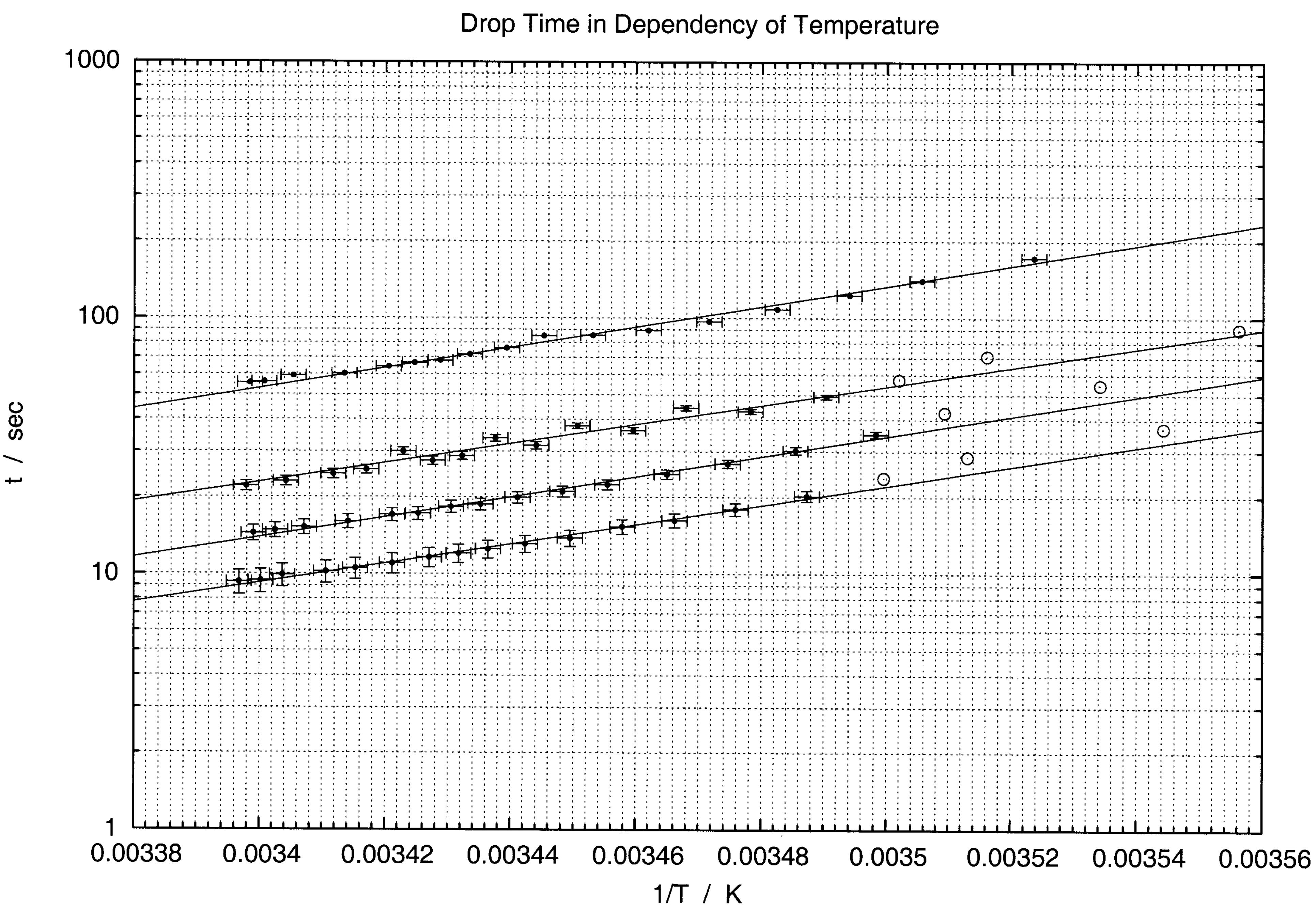
The calculation is done in the table ~~fot~~ for the balls of radius r_0

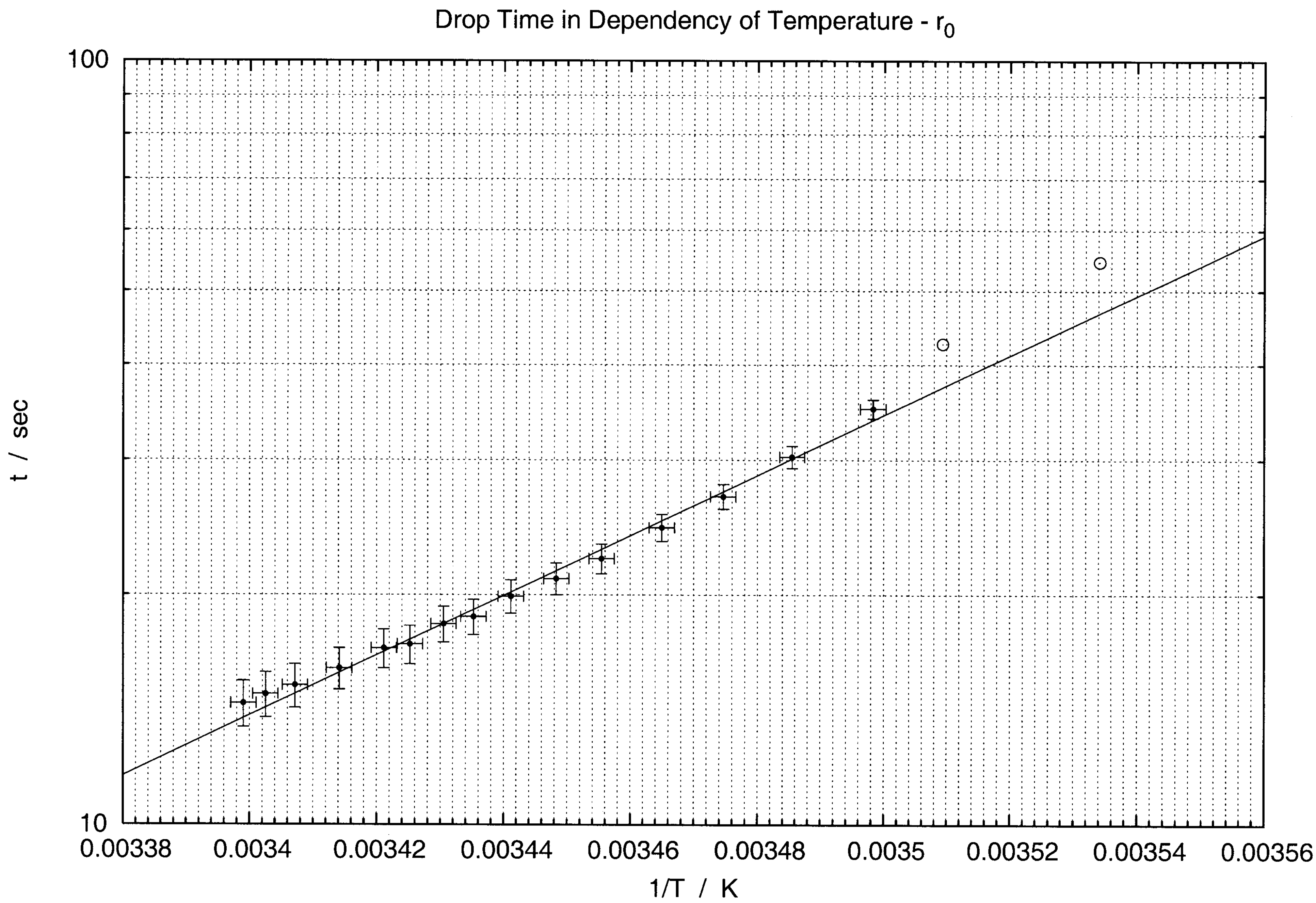
We again expect an exponential dependency:

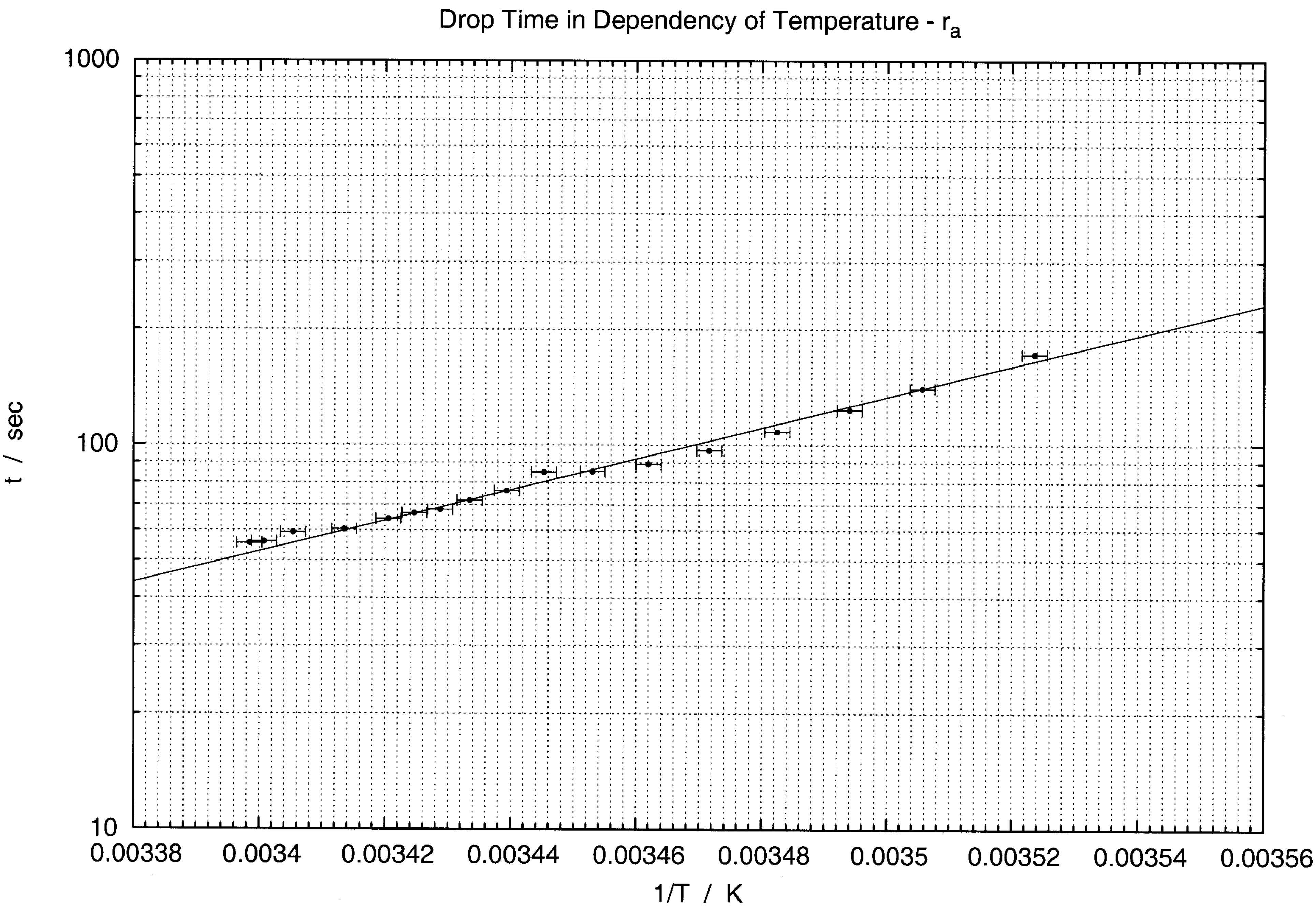
Ball-Drop-Viscosimeter: Measured Values and Calculations

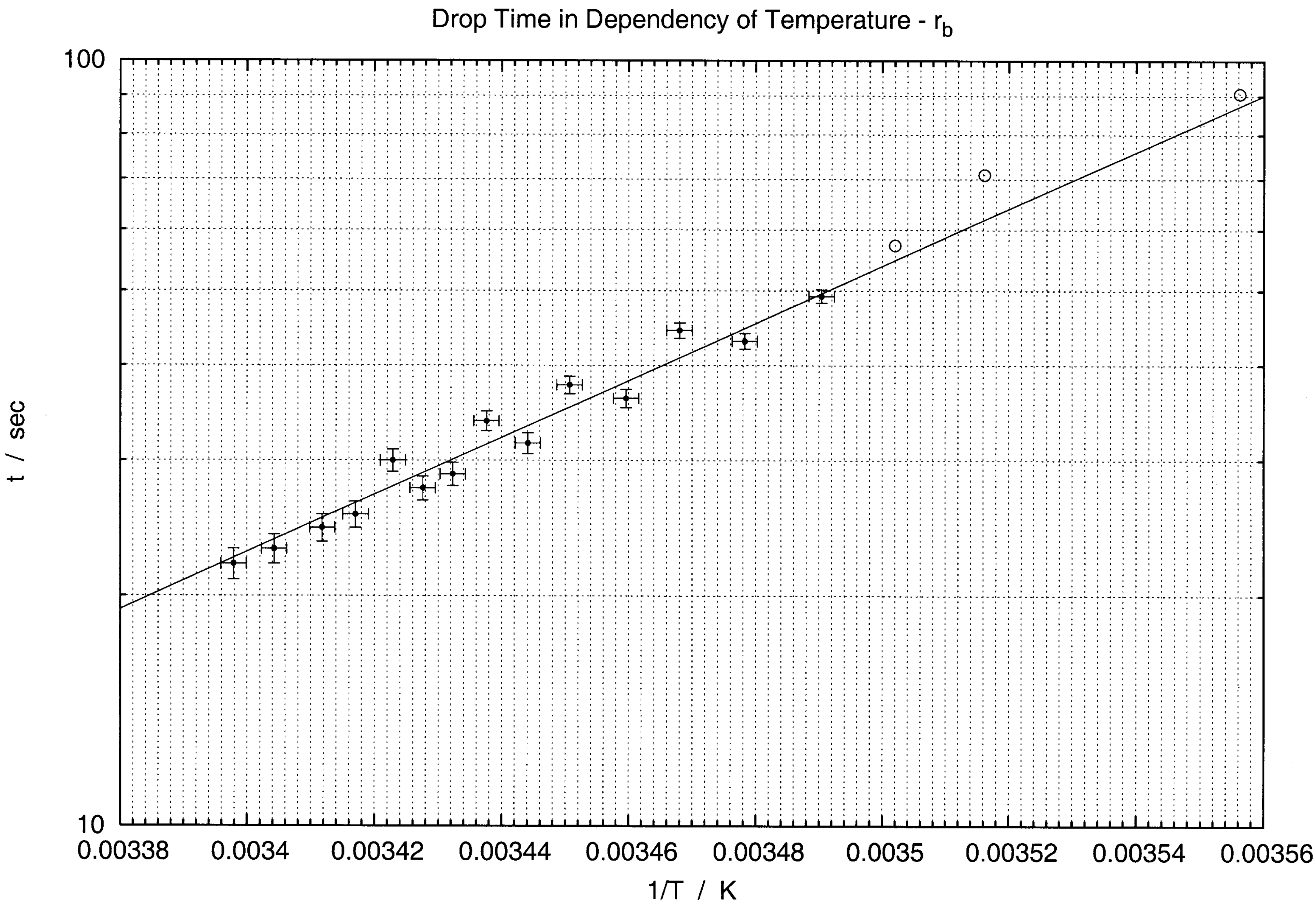
Drop distance s: 0.18 m g: 9.8128 m/s^2 Δρ: 6840 +- 30 kg/m^3 r0: 0.001 m

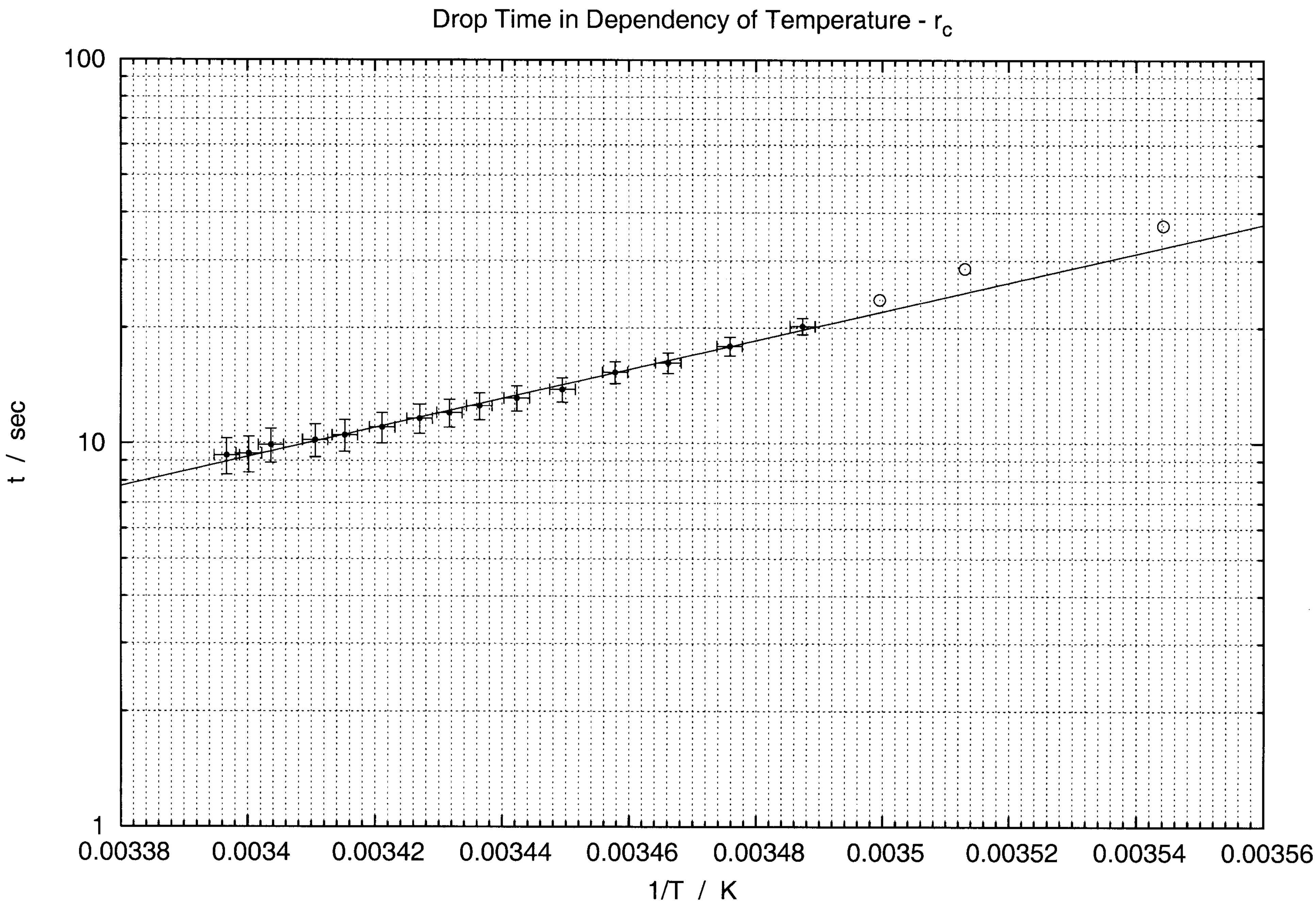
T1 / C	T2 / C	T_avg / K	error	type of ball	t / sec	error	1/T_avg	error	final veloc.	error	viscosity/ / Pa s	error	calc. radius	error	
4.7	5.9	278.5	0.1	r0	67.7	1.0	3.59E-003	2.00E-006	0.00266	0.00004	5.61	0.09			
9.6	9.9	282.95	0.1	r0	54.6	1.0	3.53E-003	2.00E-006	0.00330	0.00006	4.52	0.09			
11.7	11.8	284.95	0.1	r0	42.6	1.0	3.51E-003	2.00E-006	0.00423	0.00010	3.53	0.08			
12.6	12.7	285.85	0.1	r0	35.0	1.0	3.50E-003	2.00E-006	0.00514	0.00015	2.90	0.08			
13.7	13.7	286.9	0.1	r0	30.3	1.0	3.49E-003	2.00E-006	0.00594	0.00020	2.51	0.08			
14.6	14.6	287.8	0.1	r0	26.9	1.0	3.47E-003	2.00E-006	0.00669	0.00025	2.23	0.08			
15.4	15.4	288.6	0.1	r0	24.5	1.0	3.47E-003	2.00E-006	0.00735	0.00030	2.03	0.08			
16.2	16.2	289.4	0.1	r0	22.3	1.0	3.46E-003	2.00E-006	0.00807	0.00036	1.85	0.08			
16.8	16.8	290	0.1	r0	21.0	1.0	3.45E-003	2.00E-006	0.00857	0.00041	1.74	0.08			
17.4	17.4	290.6	0.1	r0	19.9	1.0	3.44E-003	2.00E-006	0.00905	0.00045	1.65	0.08			
17.9	17.9	291.1	0.1	r0	18.7	1.0	3.44E-003	2.00E-006	0.00963	0.00051	1.55	0.08			
18.3	18.3	291.5	0.1	r0	18.3	1.0	3.43E-003	2.00E-006	0.00984	0.00054	1.52	0.08			
18.9	18.6	291.95	0.1	r0	17.2	1.0	3.43E-003	2.00E-006	0.01047	0.00061	1.43	0.08			
19.1	19.1	292.3	0.1	r0	17.0	1.0	3.42E-003	2.00E-006	0.01059	0.00062	1.41	0.08			
19.7	19.7	292.9	0.1	r0	16.0	1.0	3.41E-003	2.00E-006	0.01125	0.00070	1.33	0.08			
20.3	20.3	293.5	0.1	r0	15.2	1.0	3.41E-003	2.00E-006	0.01184	0.00078	1.26	0.08			
20.7	20.7	293.9	0.1	r0	14.8	1.0	3.40E-003	2.00E-006	0.01216	0.00082	1.23	0.08			
21.0	21.0	294.2	0.1	r0	14.4	1.0	3.40E-003	2.00E-006	0.01250	0.00087	1.19	0.08			
6.5	8.2	280.55	0.1	ra	225.0	1.0	3.56E-003	2.00E-006	0.00080	0.00000			5.49E-004	5.76E-006	
10.3	10.9	283.8	0.1	ra	172.4	1.0	3.52E-003	2.00E-006	0.00104	0.00001			5.63E-004	7.31E-006	
11.9	12.2	285.25	0.1	ra	140.4	1.0	3.51E-003	2.00E-006	0.00128	0.00001			5.51E-004	9.16E-006	
12.9	13.1	286.2	0.1	ra	123.3	1.0	3.49E-003	2.00E-006	0.00146	0.00001			5.33E-004	1.08E-005	
13.8	14.1	287.15	0.1	ra	108.3	1.0	3.48E-003	2.00E-006	0.00166	0.00002			5.29E-004	1.24E-005	
14.7	15.0	288.05	0.1	ra	96.9	1.0	3.47E-003	2.00E-006	0.00186	0.00002			5.27E-004	1.39E-005	
15.6	15.7	288.85	0.1	ra	89.2	1.0	3.46E-003	2.00E-006	0.00202	0.00002			5.24E-004	1.51E-005	
16.3	16.5	289.6	0.1	ra	85.4	1.0	3.45E-003	2.00E-006	0.00211	0.00002			5.11E-004	1.62E-005	
17.0	17.1	290.25	0.1	ra	85.0	1.0	3.45E-003	2.00E-006	0.00212	0.00002			4.97E-004	1.67E-005	
17.5	17.6	290.75	0.1	ra	76.1	1.0	3.44E-003	2.00E-006	0.00237	0.00003			5.11E-004	1.82E-005	
18.0	18.1	291.25	0.1	ra	71.8	1.0	3.43E-003	2.00E-006	0.00251	0.00003			5.10E-004	1.93E-005	
18.4	18.5	291.65	0.1	ra	67.9	1.0	3.43E-003	2.00E-006	0.00265	0.00004			5.19E-004	2.01E-005	
18.7	18.9	292	0.1	ra	66.5	1.0	3.42E-003	2.00E-006	0.00271	0.00004			5.09E-004	2.09E-005	
19.1	19.2	292.35	0.1	ra	64.2	1.0	3.42E-003	2.00E-006	0.00280	0.00004			5.15E-004	2.14E-005	
19.7	19.8	292.95	0.1	ra	60.3	1.0	3.41E-003	2.00E-006	0.00299	0.00005			5.15E-004	2.28E-005	
20.4	20.5	293.65	0.1	ra	59.2	1.0	3.41E-003	2.00E-006	0.00304	0.00005			5.07E-004	2.36E-005	
20.8	20.9	294.05	0.1	ra	56.0	1.0	3.40E-003	2.00E-006	0.00321	0.00006			5.14E-004	2.46E-005	avg:
21.0	21.1	294.25	0.1	ra	55.5	1.0	3.40E-003	2.00E-006	0.00324	0.00006			5.09E-004	2.50E-005	5.22E-004
7.4	8.6	281.2	0.1	rb	90.6	1.0	3.56E-003	2.00E-006	0.00199	0.00002			8.64E-004	9.08E-006	
11.1	11.3	284.4	0.1	rb	70.9	1.0	3.52E-003	2.00E-006	0.00254	0.00004			8.78E-004	1.14E-005	
12.3	12.4	285.55	0.1	rb	57.3	1.0	3.50E-003	2.00E-006	0.00314	0.00005			8.62E-004	1.43E-005	
13.2	13.4	286.5	0.1	rb	49.2	1.0	3.49E-003	2.00E-006	0.00366	0.00007			8.43E-004	1.71E-005	
14.2	14.4	287.5	0.1	rb	43.0	1.0	3.48E-003	2.00E-006	0.00419	0.00010			8.39E-004	1.96E-005	
15.1	15.2	288.35	0.1	rb	44.4	1.0	3.47E-003	2.00E-006	0.00405	0.00009			7.78E-004	2.05E-005	
15.8	15.9	289.05	0.1	rb	36.2	1.0	3.46E-003	2.00E-006	0.00497	0.00014			8.23E-004	2.38E-005	
16.6	16.6	289.8	0.1	rb	37.7	1.0	3.45E-003	2.00E-006	0.00477	0.00013			7.69E-004	2.44E-005	
17.1	17.2	290.35	0.1	rb	31.6	1.0	3.44E-003	2.00E-006	0.00570	0.00018			8.15E-004	2.75E-005	
17.7	17.7	290.9	0.1	rb	33.8	1.0	3.44E-003	2.00E-006	0.00533	0.00016			7.67E-004	2.73E-005	
18.1	18.2	291.35	0.1	rb	28.8	1.0	3.43E-003	2.00E-006	0.00625	0.00022			8.06E-004	3.05E-005	
18.5	18.6	291.75	0.1	rb	27.6	1.0	3.43E-003	2.00E-006	0.00652	0.00024			8.14E-004	3.15E-005	
18.9	19.0	292.15	0.1	rb	30.0	1.0	3.42E-003	2.00E-006	0.00600	0.00020			7.57E-004	3.11E-005	
19.4	19.5	292.65	0.1	rb	25.5	1.0	3.42E-003	2.00E-006	0.00706	0.00028			8.16E-004	3.40E-005	
19.9	19.9	293.1	0.1	rb	24.5	1.0	3.41E-003	2.00E-006	0.00735	0.00030			8.08E-004	3.57E-005	
20.5	20.6	293.75	0.1	rb	23.0	1.0	3.40E-003	2.00E-006	0.00783	0.00034			8.13E-004	3.78E-005	avg:
21.1	21.1	294.3	0.1	rb	22.0	1.0	3.40E-003	2.00E-006	0.00818	0.00037			8.20E-004	3.92E-005	8.16E-004
8.6	9.3	282.15	0.1	rc	37.0	1.0	3.54E-003	2.00E-006	0.00486	0.00013			1.35E-003	1.42E-005	
11.4	11.5	284.65	0.1	rc	28.6	1.0	3.51E-003	2.00E-006	0.00629	0.00022			1.38E-003	1.80E-005	
12.5	12.6	285.75	0.1	rc	23.7	1.0	3.50E-003	2.00E-006	0.00759	0.00032			1.34E-003	2.23E-005	
13.5	13.6	286.75	0.1	rc	20.2	1.0	3.49E-003	2.00E-006	0.00891	0.00044			1.32E-003	2.66E-005	
14.5	14.5	287.7	0.1	rc	17.9	1.0	3.48E-003	2.00E-006	0.01006	0.00056			1.30E-003	3.04E-005	
15.3	15.3	288.5	0.1	rc	16.2	1.0	3.47E-003	2.00E-006	0.01111	0.00069			1.29E-003	3.39E-005	
16.0	16.0	289.2	0.1	rc	15.3	1.0	3.46E-003	2.00E-006	0.01176	0.00077			1.27E-003	3.65E-005	
16.7	16.7	289.9	0.1	rc	13.8	1.0	3.45E-003	2.00E-006	0.01304	0.00095			1.27E-003	4.03E-005	
17.3	17.3	290.5	0.1	rc	13.1	1.0	3.44E-003	2.00E-006	0.01374	0.00105			1.27E-003	4.27E-005	
17.8	17.8	291	0.1	rc	12.5	1.0	3.44E-003	2.00E-006	0.01440	0.00115			1.26E-003	4.49E-005	
18.2	18.2	291.4	0.1	rc	12.0	1.0	3.43E-003	2.00E-006	0.01500	0.00125			1.25E-003	4.72E-005	
18.6	18.6	291.8	0.1	rc	11.6	1.0	3.43E-003	2.00E-006	0.01552	0.00134			1.26E-003	4.86E-005	
19.1	19.1	292.3	0.1	rc	11.0	1.0	3.42E-003	2.00E-006	0.01636	0.00149			1.25E-003	5.14E-005	
19.6	19.6	292.8	0.1	rc	10.5	1.0	3.42E-003	2.00E-006	0.01714	0.00163			1.27E-003	5.29E-005	
20.0	20.0	293.2	0.1	rc	10.2	1.0	3.41E-003	2.00E-006	0.01765	0.00173			1.25E-003	5.54E-005	
20.6	20.6	293.8	0.1	rc	9.9	1.0	3.40E-003	2.00E-006	0.01818	0.00184			1.24E-003	5.77E-005	
20.9	20.9	294.1	0.1	rc	9.4	1.0	3.40E-003	2.00E-006	0.01915	0.00204			1.25E-003	6.00E-005	avg:
21.2	21.2	294.4	0.1	rc	9.3	1.0	3.40E-003	2.00E-006	0.01935	0.00208			1.24E-003	6.11E-005	1.28E-003


Drop Time in Dependency of Temperature
t / sec
1000
100
10
1
0.00338
0.0034
0.00342
0.00344
0.00346
0.00348
0.0035
0.00352
0.00354
0.00356
1/T / K

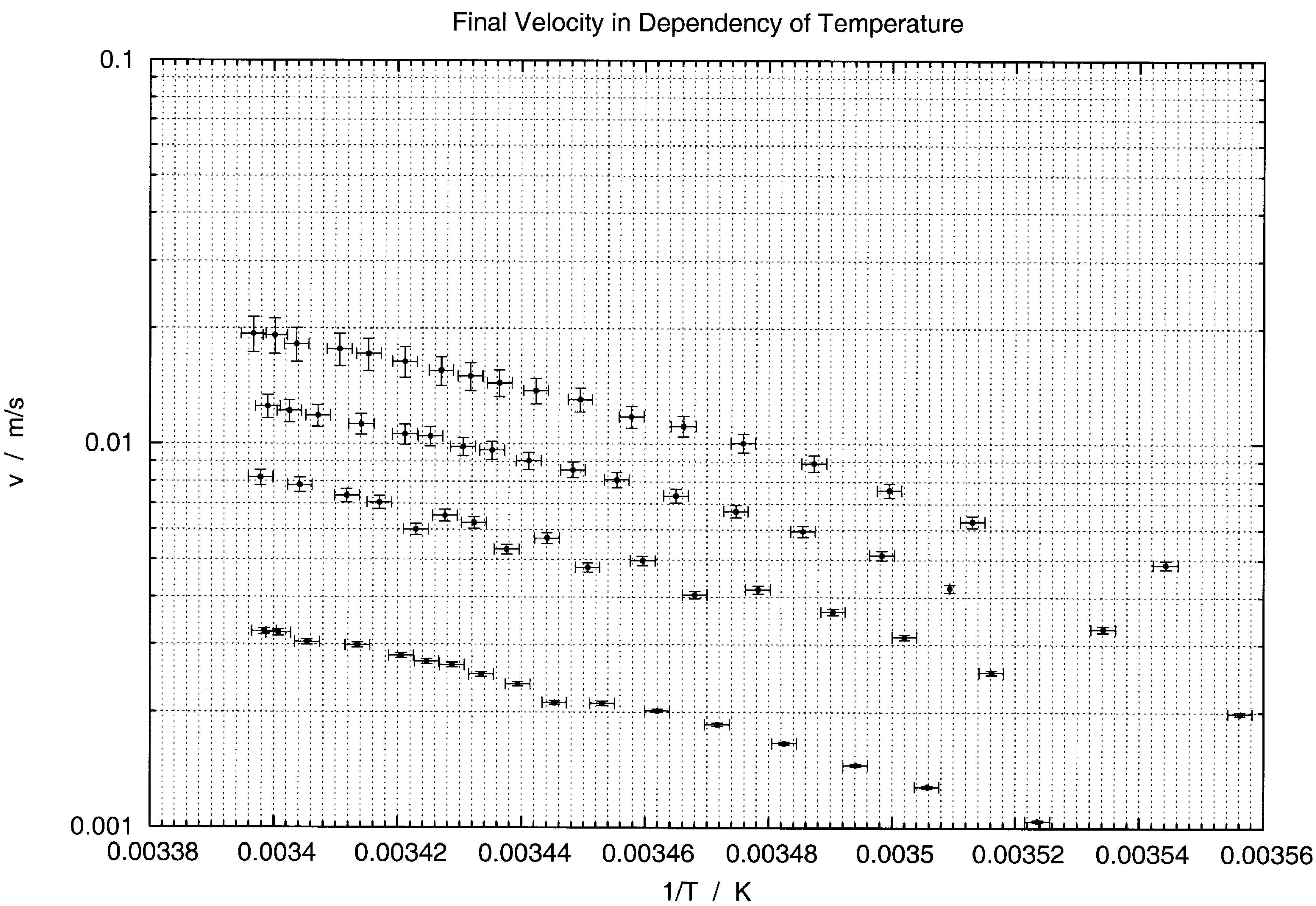

Drop Time in Dependency of Temperature - r0
100
10
t / sec
0.00338
0.0034
0.00342
0.00344
0.00346
0.00348
0.0035
0.00352
0.00354
0.00356
1/T / K

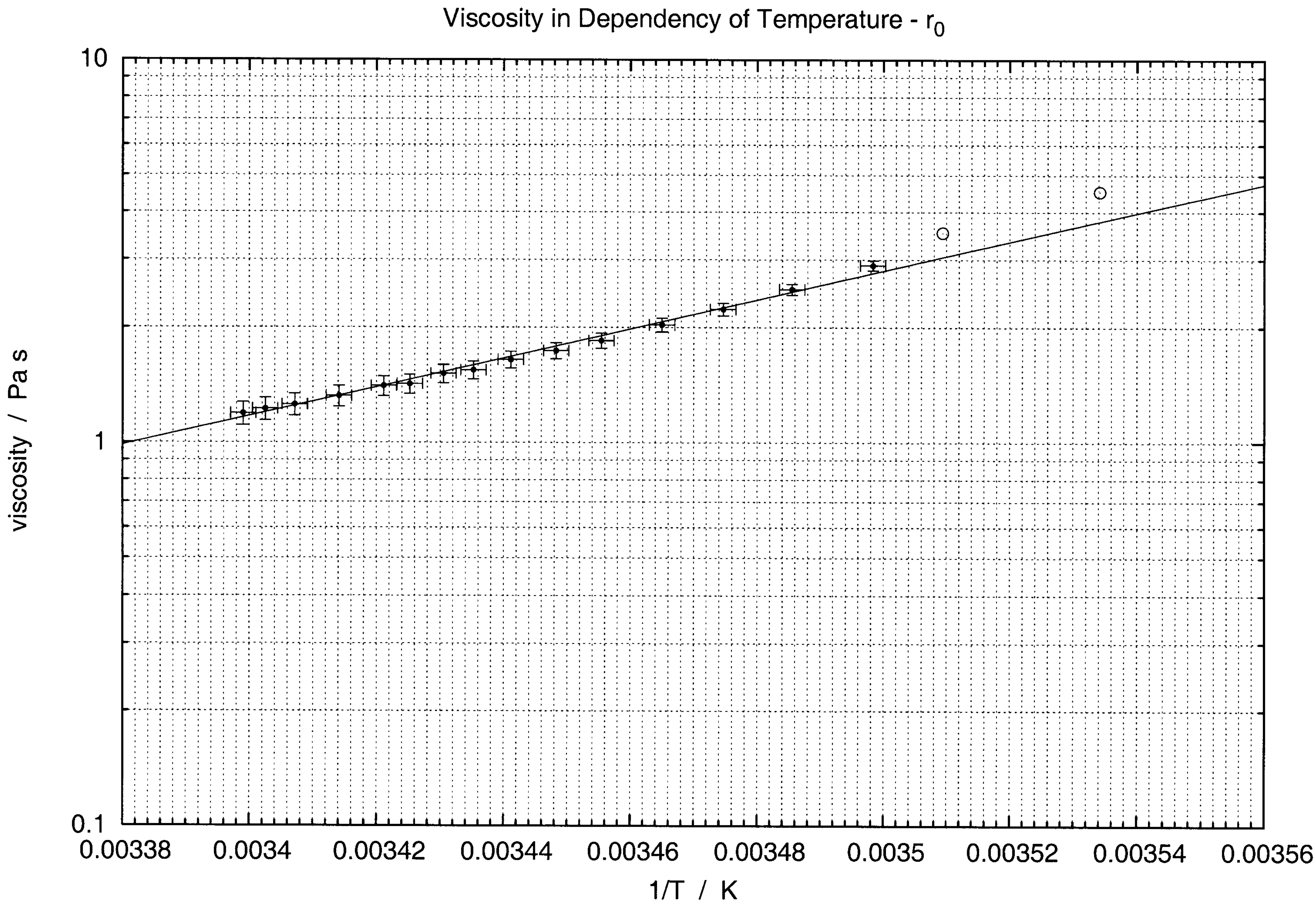

Drop Time in Dependency of Temperature - r_a
1000
100
10
t / sec
0.00338
0.0034
0.00342
0.00344
0.00346
0.00348
0.0035
0.00352
0.00354
0.00356
1/T / K


Drop Time in Dependency of Temperature - r_b
t / sec
100
10
0.00338
0.0034
0.00342
0.00344
0.00346
0.00348
0.0035
0.00352
0.00354
0.00356
1/T / K



Drop Time in Dependency of Temperature - r_c
t / sec
100
10
1
0.00338
0.0034
0.00342
0.00344
0.00346
0.00348
0.0035
0.00352
0.00354
0.00356
1/T / K


Final Velocity in Dependency of Temperature
0.1
0.01
0.001
v / m/s
0.00338
0.0034
0.00342
0.00344
0.00346
0.00348
0.0035
0.00352
0.00354
0.00356
1/T / K


Viscosity in Dependency of Temperature - r_0
viscosity / Pa s
10
1
0.1
0.00338
0.0034
0.00342
0.00344
0.00346
0.00348
0.0035
0.00352
0.00354
0.00356
1/T / K

$$\eta(T) = A \cdot e^{B/T}$$

The plot fully satisfies this expectation with $B = (8717 \pm 232)$ Pa s (asymptotic st. error)
The value for A was determined as $1{,}58 \cdot 10^{-13}$ Pa·s with a large error close to 100%
With this fit, the viscosity at room temperature = 20°C can be read off as 1,29 Pa·s. This value also has a large error due to the uncertainty of B. The literature value of 0,985 Pa s might well be in one error intervall

Assignment 3:

From $\eta(T) = \frac{2 g r^2 \Delta\rho}{9 s} \cdot t(T)$

follows that

$$\frac{r^2 t}{s} = \text{const.}$$

So, we can calculate the radius of an unknown ball as

$r_x = r_0 \cdot \sqrt{\frac{t_0}{t_x}}$, t_0 and t_x being at the same temperature

The table shows the calculation for each measurement and the average radius for each ball. The statistical error of this average value is not within significant range
In comparison with the directly measured

values we get the following table

	direct measurement	indirect measurement
r_a	$(0,725 \pm 0,001)$ mm	$(0,522 \pm 0,001)$ mm
r_b	$(0,760 \pm 0,001)$ mm	$(0,816 \pm 0,001)$ mm
r_c	$(1,235 \pm 0,001)$ mm	$(1,28 \pm 0,001)$ mm

The value for r_a can be neglected; because not all balls labeled r_a had the same radius. The other two values are significantly different from ~~to~~ the direct measurement, but in this small error interval a number of systematic errors have to be taken into account, such as the ~~unbalanced~~ inhomogeneous heat distribution within the oil.
The indirect measurement appears not as a useful technique to determine the balls' radii.

Assignment 4

The differential equation for the law of movement was already stated and solved in the preparation
The time after which the ball reaches 99,9% of its final velocity is

$$t = \frac{-2\rho_K \cdot r^2}{9\eta} \cdot \ln(0,001)$$

With the literature values

$\varrho_{K} = 7{,}81 \cdot 10^{3}$ kg/m³ and

$\eta = 0{,}985$ Pa·s at 70°C

we find

radius	t / sec
$r_0 = 1 \cdot 10^{-3}$ m	0,012
$r_a = 0{,}725 \cdot 10^{-3}$ m	0,007
$r_b = 0{,}76 \cdot 10^{3}$ m	0,007
$r_c = 1{,}235 \cdot 10^{-3}$ m	0,019

Obviously, the time and thus the path after which the balls reach constant speed is very small and does not have any effect on the experiment.

Conclusion

In general, the experiment showed the exspected exponential dependencies of the various quantities on the temperature very nicely.
For specific values, however, the experiment did not provide the necessary accuracy to have number in agreement with the literature values. Systematic errors and the high reading error for exponential plots made most ~~values~~ specific results unusable.
Still, the experiment can be considered successful, as all the theoretical laws were met quantitatively.

Another probable cause of error is dirt on the bulbs or in the oil.

(5)